General Stuff

- Office Hours: Today Tuesdays after class 12:30 - 1:30, Thursday before class 10 - 11am
- Lab 0 Due tonight
- Quiz parameters (1/28 Quiz)

  15 min + 5 min to upload Scanning apps or picture

  *At the end of class on Thursday Start at 11:45

  No notes or calculators

  Cameras TURNED ON until you've uploaded to gradescope and checked in with me

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Quizzes + Midterms materials week (or 2 weeks) before

Quiz 1 is on 1.3

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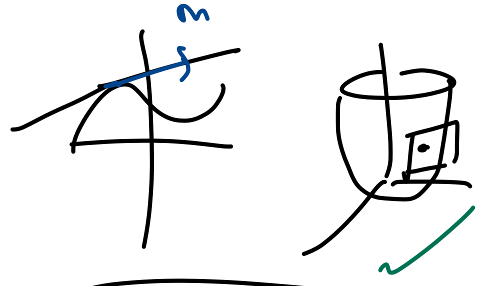
Review

- How to make a plane

  Cartesian equation (normal vector + constant)

  Parametrization (two direction vectors + point in the plane)

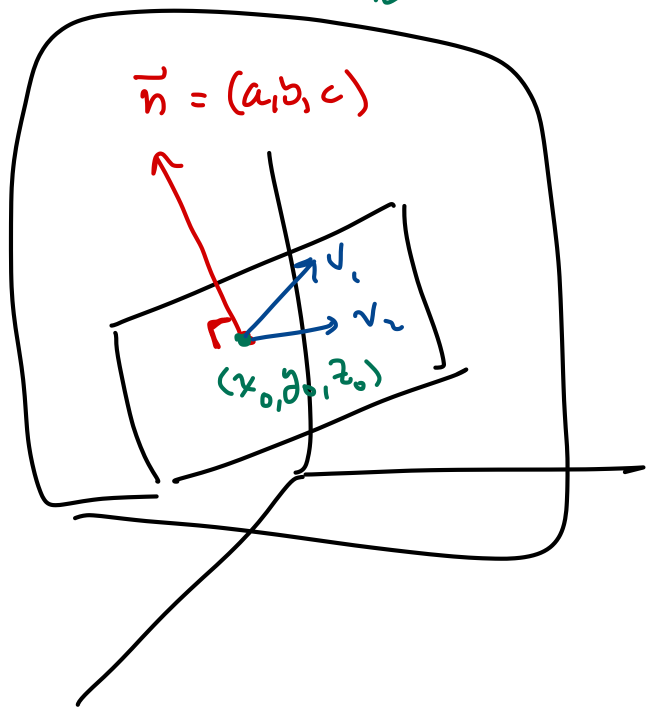


Cartesian equation

$$ax + by + cz = d$$

$\vec{n} = (a,b,c)$

normal vector



$d = (a,b,c)\cdot(x_0,y_0,z_0)$

where (x_0,y_0,z_0) is some point in the plane

$$p(s,t) = \underline{(x_0,y_0,z_0)} + s\vec{v}_1 + t\vec{v}_2$$

direction vectors

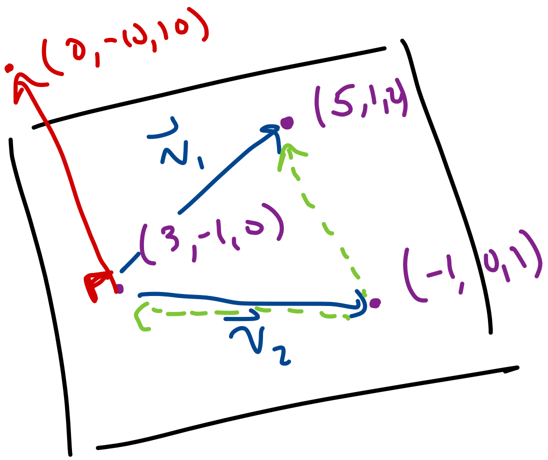
$$\vec{n} = \vec{v}_1 \times \vec{v}_2$$

1. Find the equation of the plane that contains the three points ~~(0, 1, 3)~~, ~~(1, 1, 0)~~, and ~~(3, 0, −1)~~.

$(5, 1, 2)$, $(3, -1, 0)$, $(-1, 0, 1)$.

$$ax + by + cz = d$$

$n = (a, b, c)$ = normal $= \vec{v}_1 \times \vec{v}_2$

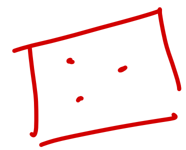


$$\vec{v}_1 = (5,1,2) - (3,-1,0)$$
$$= (2, 2, 2)$$

(order doesn't matter)

$$\vec{v}_2 = (-1,0,1) - (3,-1,0)$$
$$= (-4, 1, 1)$$

In general: 3 points determine a plane.



$$n = (2,2,2) \times (-4,1,1) = \det\begin{bmatrix} i & j & k \\ 2 & 2 & 2 \\ -4 & 1 & 1 \end{bmatrix} = \big(2\cdot 1 - 2\cdot 1,\ -(2\cdot 1 - -4\cdot 2),\ 2\cdot 1 - (-4)\cdot 2\big)$$

$$= (\underset{a}{0},\ \underset{b}{-10},\ \underset{c}{10})$$

$0\cancel{x} - 10y + 10\cancel{z} = d$

$d = (0, -10, 10) \cdot (x_0, y_0, z_0)$

$\uparrow$ Some point in the plane

We'll pick $(5,1,2)$

Or plug in point solve for d

$d = (0, -10, 10) \cdot (5, 1, 2)$

$= 0 \cdot 5 + (-10) \cdot 1 + 10 \cdot 2$

$= -10 + 20 = 10$

$0x - 10y + 10z = 10$

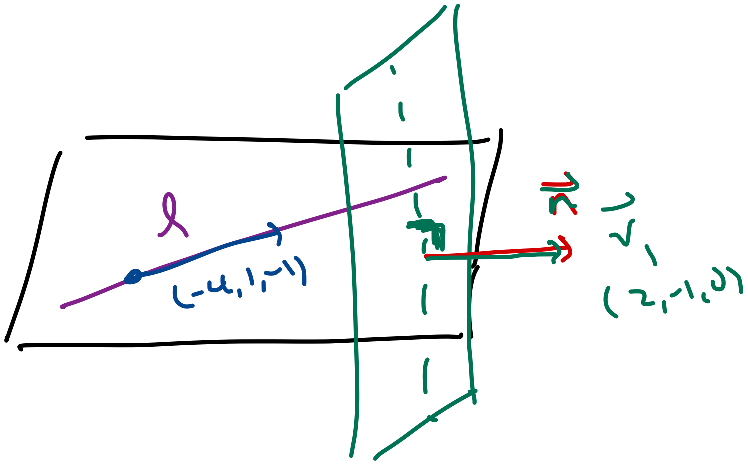
$$\boxed{-y + z = 1}$$

$(5,1,2)$, $(3,-1,0)$, $(-1,0,1)$

$-1+2=1$ $\quad -(-1)+0=1$ $\quad 0+1=1$

2. Find the equation of the plane which contains the line $\ell(t) = (-1, 0, 1) + t(-4, 1, -1)$ and is perpendicular to $2x - y = 3$.

point in the plane



$ax + by + cz = d$

$n = (a, b, c)$ $\qquad d = n \cdot (x_0, y_0, z_0)$

$n = \vec v_1 \times \vec v_2$

$\vec n$ for $2x - y = 3$ is a direction vector for our mystery plane

$\cancel{n} = (2, -1, 0) = \vec v_1$

v_2 is the direction of the line aka $v_2 = (-4, 1, -1)$

$-(2(-1) - (-4)\cdot 0)$

$-(-2) = 2$

$$(a,b,c) = n = (2,-1,0) \times (-4,1,-1) = \begin{array}{ccc} + & - & + \end{array} \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ -4 & 1 & -1 \end{vmatrix} = (1, 2, -2)$$

$x + 2y - 2z = d$ $\qquad d = -1 + 2(0) - 2(1) = -3$

$$\boxed{x + 2y - 2z = -3}$$

10 11:55

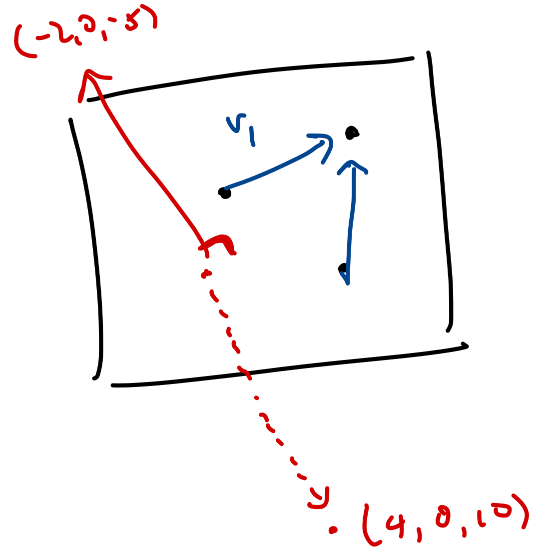
Take ~~15~~ minutes to work on the following problems.

* **3.** Find the equation of the plane which contains the 3 points $(-3, 1, 1)$, $(2, 1, -1)$, and $(0, 0, 1)$.

* **4.** Find the equation of the plane containing the two lines $\ell_1(t) = (0, 2, 0) + t(-1, 2, 0)$ and $\ell_2(t) = (1, 0, 0) + t(0, 3, 1)$.

5. Find the parametrization of the line which is the intersection of the planes $x + y - z = 2$ and $-2x + 3y - z = 3$.

3. Find the equation of the plane which contains the 3 points $(-3, 1, 1)$, $(2, 1, -1)$, and $(0, 0, 1)$.



$$v_1 = (-3,1,1) - (2,1,-1) = (-5,0,2)$$

$$v_2 = (-3,1,1) - (0,0,1) = (-3,1,0)$$

$$n = (a,b,c) = (-5,0,2) \times (-3,1,0)$$

$$= \begin{vmatrix} i & j & k \\ -5 & 0 & 2 \\ -3 & 1 & 0 \end{vmatrix} = (-2,-6,-5)$$

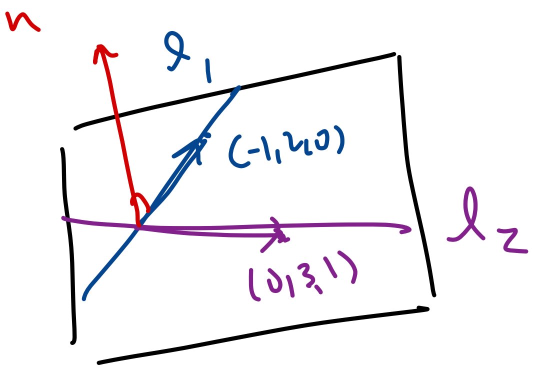
$$-\left(-5 \cdot 0 - (2)\cdot(-3)\right)$$

$$-\left(-(-6)\right) = -6$$

$$d = (-2,-6,-5)\cdot(0,0,1) = -5$$

$$-2x - 6y - 5z = -5 \xrightarrow{-2} 4x + 12y + 10z = 10$$

4. Find the equation of the plane containing the two lines $\ell_1(t) = (0,2,0) + t(-1,2,0)$ and $\ell_2(t) = (1,0,0) + t(0,3,1)$.



The direction vectors of the lines are the direction vectors of the plane.

$$(a,b,c) = n = (0,3,1) \times (-1,2,0)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 1 \\ -1 & 2 & 0 \end{vmatrix}$$

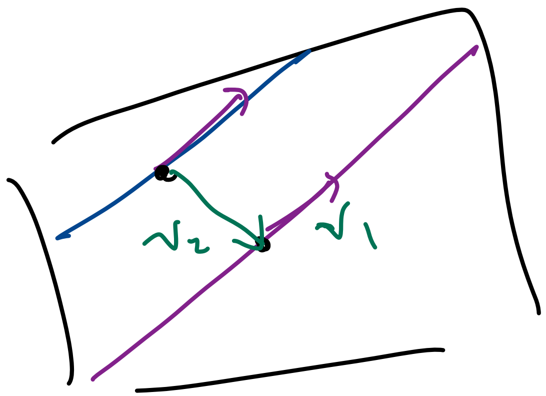
$$= (-2,-1,3) \qquad d = (-2,-1,3)\cdot(0,2,0)$$

$$= -2$$

$$-2x - y + 3z = -2$$

$$\ell_1(t) = (0,2,0) + t\underbrace{(-1,2,0)}$$

$$\ell_2(t) = (3,5,2) + t\underbrace{(-1,2,0)}$$



$$v_1 = (-1,2,0)$$

$$v_2 = \ ??$$

$$v_2 = (3,5,2) - (0,2,0)$$

$$= (3,3,2)$$

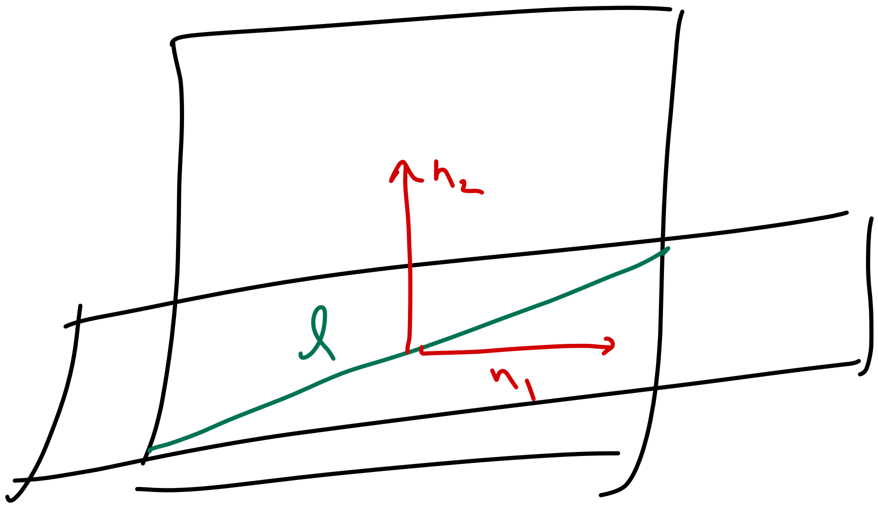
$n = v_1 \times v_2$ etc...

5. Find the parametrization of the line which is the intersection of the planes $x + y - z = 2$ and $-2x + 3y - z = 3$.

$(1, 1, -1)$

$(-2, 3, -1)$

$\ell = b_0 + t\vec{v}_1$



The line ℓ is $\perp$ to both normals, $n_1 = (1,1,-1)$ and $n_2 = (-2,3,-1)$

So the direction vector for ℓ is

$$v_1 = (1,1,-1) \times (-2,3,-1) = (2,3,5).$$

Now we need any basepoint for ℓ, aka a point in both planes. This is linear algebra.

$$x + y - \cancel{z} = 2$$
$$-\quad -2x + 3y - \cancel{z} = 3$$
$$3x - 2y = -1 \qquad y = \frac{-1-3x}{2}$$

If $x = 1$ $y = -2$

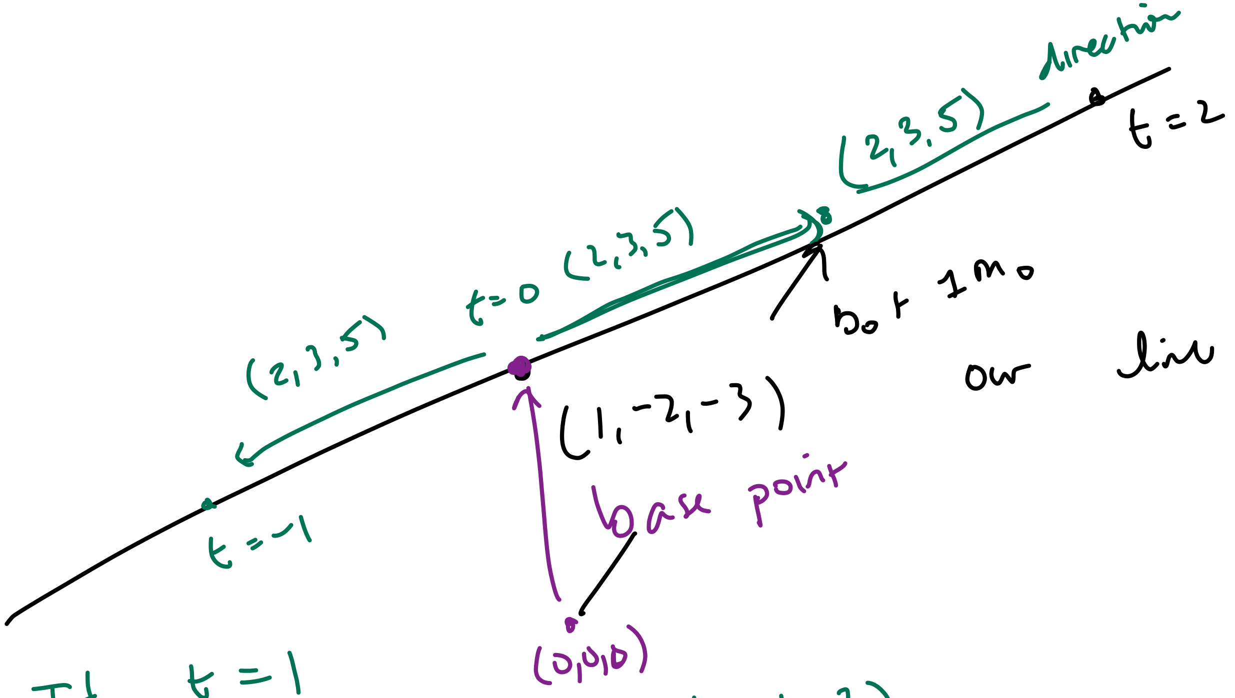
$1 - 2 - z = 2$

$z = -3$

So $b_0 = (1,-2,-3)$ and

$$\boxed{\ell(t) = (1,-2,-3) + t(2,3,5)}$$

$$\ell(t) = \underline{\overset{b}{(1,-2,-3)}} + \underline{\overset{x}{t}\overset{m}{(2,3,5)}}$$ slope



our line goes through (1,-2,-3)

↓

y intercept

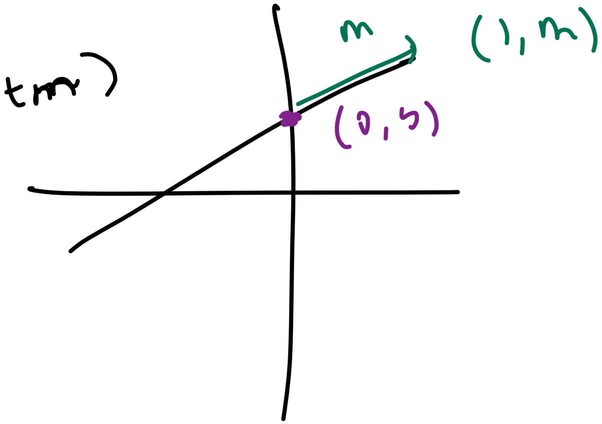
If $t = 1$

$$(1,-2,-3) + (2,3,5) = (3,1,2)$$

t

↓

$y = mx+b$



$$(x,y) = \boxed{(0,b)} + t\boxed{(1,m)} = (t, b+tm)$$

$x = t \qquad y = b+tm$

$\rightarrow \quad y = mx+b$