General Stuff

- Lab 1 today after Discussion 12:20 - 1:10   1, 3, 6   OH: T after class
                                                            Th before class
- Quiz 1 today at 11:45

    15 minutes to do the quiz   12:00

    5 minutes to upload   12:05 → Have your camera on!! (Just for proctoring.)

- Any questions on quiz material (1.3)?

    Cross products

    Determinants

    Finding equations of planes

    Parametrizations of lines or planes

Parametrization of a plane.

Given 2 direction vectors $\vec{v}_1, \vec{v}_2$, and base point b_0,

$$\boxed{p(s,t) = \underbrace{b_0 + s\vec{v}_1}_{\text{line}} + \underbrace{s\vec{v}_1 + t\vec{v}_2}_{\text{2D worth of direction vectors}}}$$

Ex Let P be the plane $\underline{x - y + z = 1}$. Find a parametrization of P.

$(1,-1,1) = n$

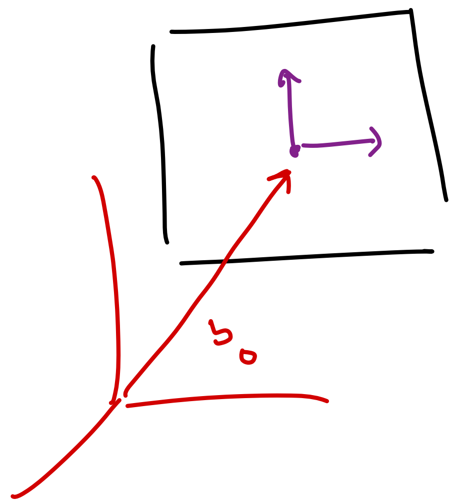
Need: direction vectors $\vec{v}_1$, $\vec{v}_2$

base point b_0. $\leftarrow$ any point on the plane.

$x = 0, y = 0$, solve for z $\qquad b_0 = (0,0,z)$

$0 - 0 + z = 1$

$z = 1 \qquad b_0 = (0,0,1)$



$v_1 \perp n = (1,-1,1)$

$v_2 \perp n = (1,-1,1)$

$\iff$

$(1,-1,1) \cdot \vec{v}_1 = 0$

$(1,-1,1) \cdot \vec{v}_2 = 0$

$v_1 = (a,b,c)$

$v_2 = (d,e,f)$

$(1,-1,1)\cdot(a,b,c) = 0$ $\quad a - b + c = 0$ $\quad (a,b,c) = (1,1,0)$

$(1,-1,1)\cdot(d,e,f) = 0$ $\quad d - e + f = 0$ $\quad (d,e,f) = (0,1,1)$

Choice, find any two solutions

inf. choices

ok, planes have an ∞ amount of directions, we just need 2

$$P(s,t) = (0,0,1) + s(1,1,0) + t(0,1,1)$$

$1x \overset{-1}{-} y + \overset{1}{1}z = 1$ $\quad$ System w/ 1 eq 3 variables

$$\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

RREF $\quad$ y free $\quad y = s$

z free $\quad z = t$

$$x = s - t + 1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s-t+1 \\ s \\ t \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\text{base point}} + s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

direction vectors

$d = \vec{n} \cdot \vec{x}_0$ where $\vec{x}_0$ is any point in the plane.

<u>Ex</u> If $\vec{n} = (2, -3, 1)$ $2x - 3y + z = d$

and $\vec{x}_0 = (5, 1, 1)$ is a point on the plane.

$d = (2, -3, 1) \cdot (5, 1, 1) = 10 - 3 + 1 = 8$

same

OR plug in $(x, y, z) = (5, 1, 1)$ and solve for d

$2 \cdot 5 - 3 \cdot 1 + 1 \cdot 1 = d = 8$

$\Longrightarrow \quad 2x - 3y + z = 8$

Review

- Equation for minimum distance from a point (x_1, y_1, z_1) to a plane $ax + by + cz = d$.

$$\text{dist} = \frac{\left|ax_1 + by_1 + cz_1 \overset{-}{\cancel{+}} d\right|}{\sqrt{a^2 + b^2 + c^2}}$$

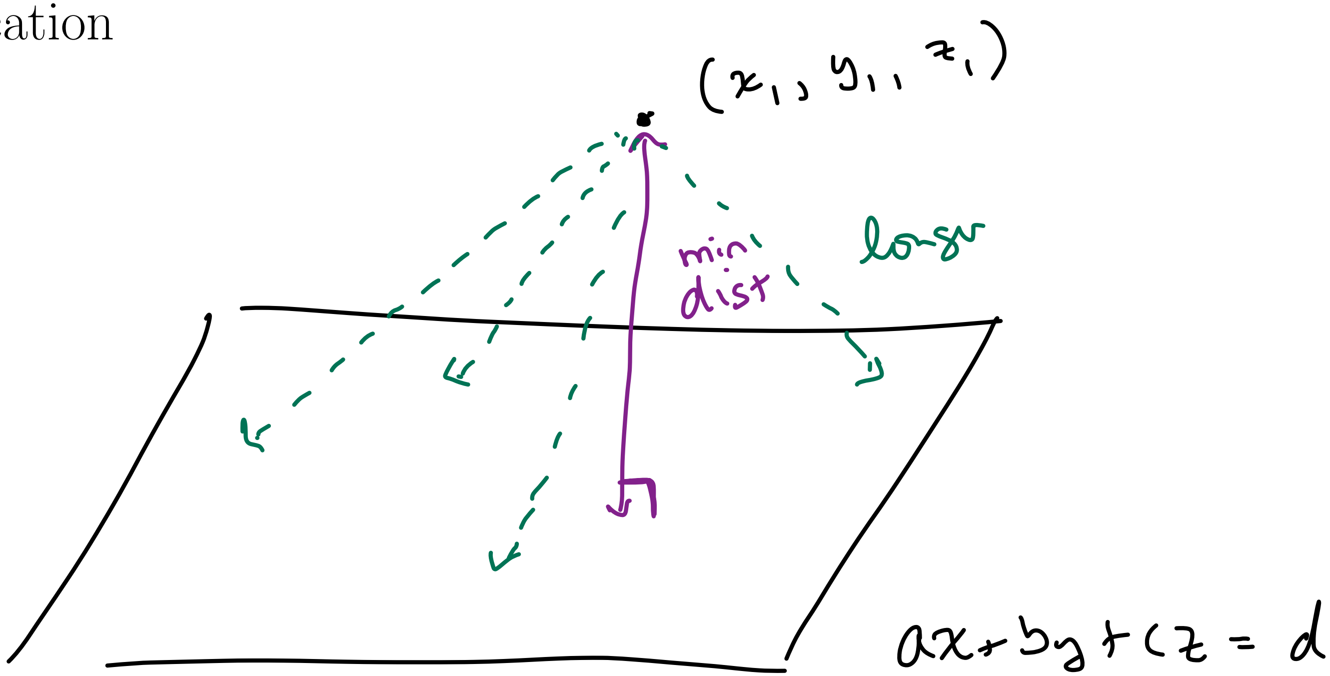
- Vector properties of $\mathbb{R}^n$

  dot product

  magnitude

  vector addition and scalar multiplication

  Matrix multiplication



1. Find the minimum distance from the point $p = (-1, 2, 0)$ to the plane $x - y + 2z = 3$. $\leftarrow d$

$$\underset{x_1}{-1},\ \underset{y_1}{2},\ \underset{z_1}{0}$$

$(a, b, c) = n$

$$n = (\underset{a}{1}, \underset{b}{-1}, \underset{c}{2})$$

$d = 3$

$$\text{dist} = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|1 \cdot (-1) + (-1) \cdot 2 + 2 \cdot (0) - 3|}{\sqrt{1^2 + (-1)^2 + 2^2}}$$

$$= \frac{|-1 - 2 - 3|}{\sqrt{6}} = \frac{|-6|}{\sqrt{6}} = +\sqrt{6}$$

$$\det \begin{bmatrix} - & \cdot & - \\ \cdot & \cdot & - \\ \cdot & - & \cdot \end{bmatrix} < 0 \implies \text{left handed set of column vectors}$$

2. Let

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -6 & 1 \\ 4 & 0 \end{pmatrix} \quad v = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}.$$

Find which combinations of A, B, and v can be multiplied and evaluate them.

We usually denote functions with arrows. The notation

$$T : \mathbb{R}^n \to \mathbb{R}^m$$

means the function is called T, the domain is $\mathbb{R}^n$, and the codomain is $\mathbb{R}^m$. Multiplying by an $m \times n$ matrix is a function like this. For example...

Functions where you multiply by a matrix are called *linear* transformations because...

Derivatives in multi will be linear transformations. We can already see that a bit using the parametrization of a plane.