General Stuff

- Office Hours

  T: 12:30 - 1:30, Th: 10 - 11

- Lab 4 due tonight

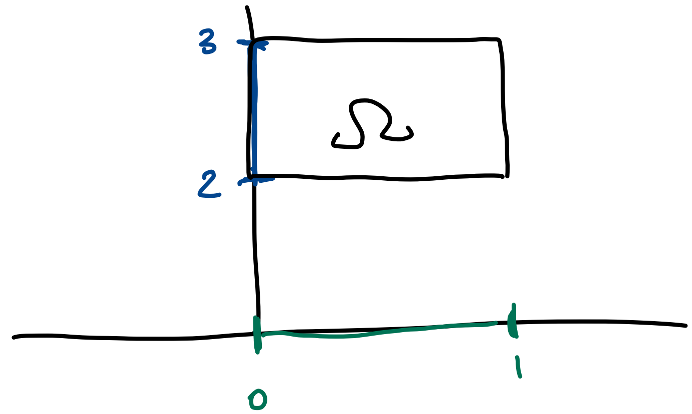
  Exercises 1,3 – same setup, so just do them together

- Quiz Thursday 5.1 – 5.2 double integrals

1. Evaluate the integral

$$\iint_{\Omega} x^2y + xy^2 \, dx \, dy$$

where $\Omega = \underbrace{[0,1]}_{x} \times \underbrace{[2,3]}_{y}$.



$$\iint_{\Omega} x^2y + xy^2 \, dx \, dy$$

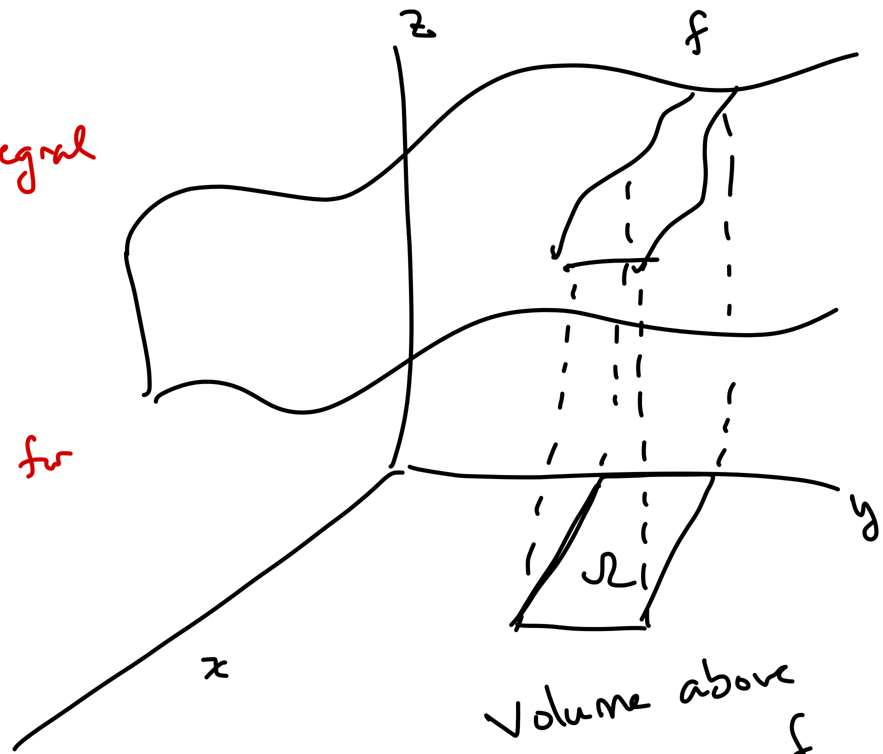
$$= \int_2^3 \int_0^1 x^2y + xy^2 \, dx \, dy$$

y is a constant in the dx integral

$$= \int_2^3 \left(\frac{1}{3}x^3y + \frac{1}{2}x^2y^2 \right)\Big|_0^1 dy$$

plug in 1, 0 for x

$$= \int_2^3 \left(\frac{1}{3}(1)^3y + \frac{1}{2}(1)^2y^2 \right) - \underbrace{\cancel{\left(\frac{1}{3}0^3y + \frac{1}{2}0^2y^2 \right)}}_{0} dy$$



Volume above Ω, below f is this integral!

at this point no more x's

$$= \int_2^3 \frac{1}{3} y + \frac{1}{2} y^2 \, dy = \left(\frac{1}{6} y^2 + \frac{1}{6} y^3 \right)_2^3$$

$$= \frac{1}{6} \left((3^2 + 3^3) - (2^2 + 2^3) \right)$$

$$= \frac{1}{6} \left(9 + 27 - 4 - 8 \right) = \frac{24}{6} = \boxed{4}$$

volume above Ω below $f = x^2 y + x y^2$

2. Let W be the region bounded by the equations $x \cancel{=} 0$ $x \geq 0$, $y = 0$, $y = 1$ and $y = x^2$. Evaluate the integral

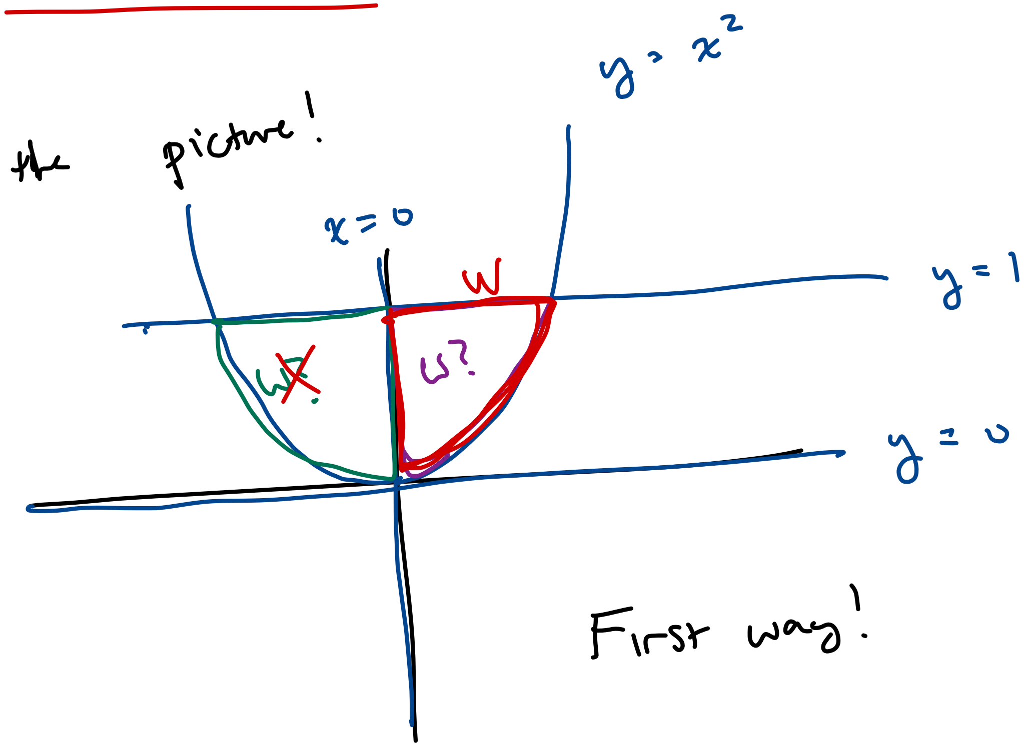
$$\iint_W x - y \, \boxed{dA}$$

either dxdy or dydx

we'll do it both ways to practice but normally 1 of the orders is easier than the other.

using two different orders.

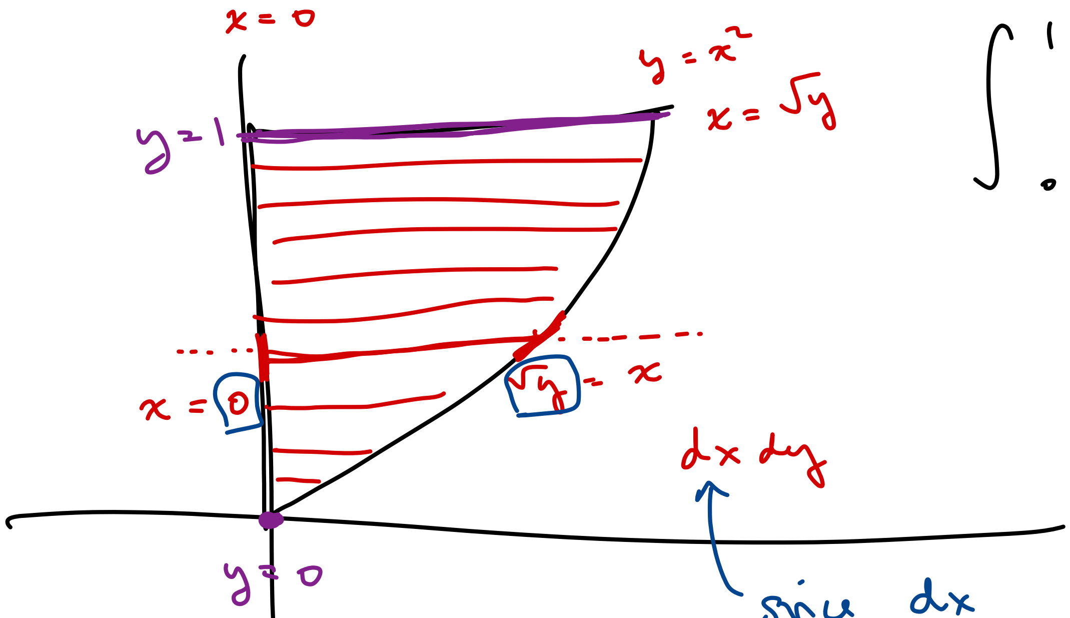
Draw the picture!



First way!

$$\iint_W x - y \, dA = \iint_W x - y \, dx \, dy$$

Cut horizontal!



since dx is first, cut region horizontally!

$$\int_0^1 \int_0^{x^2 \ \sqrt{y}} x - y \, \boxed{dx} \, dy$$

only y's in the bounds!

$$\int_0^1 \left(\frac{1}{2}x^2 - xy \right)\Big|_0^{x^2} dy$$

$$\int_0^1 \left(\frac{1}{2}x^4 - x^2 y \right) dy$$

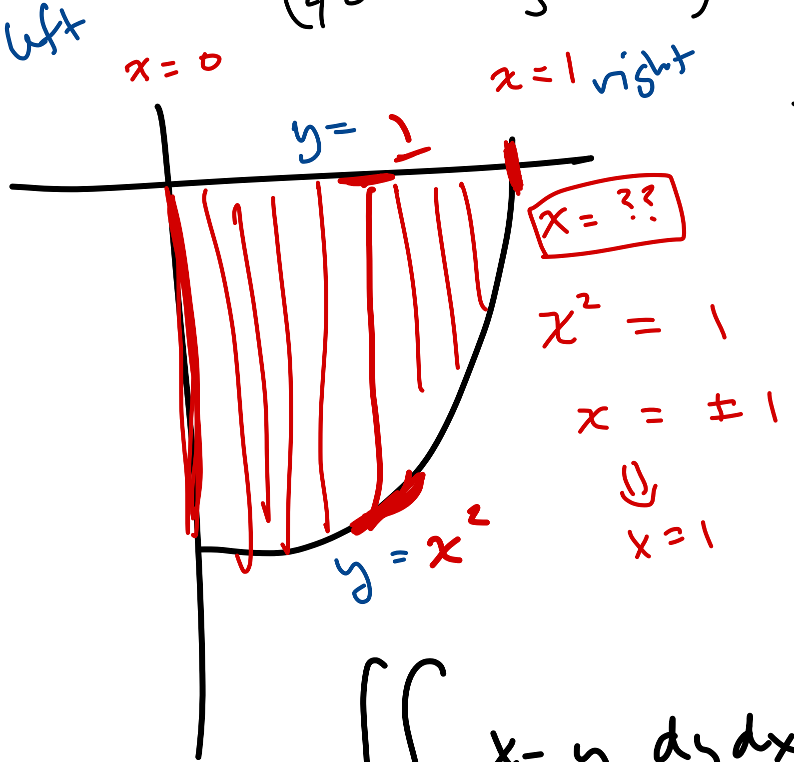
BAD

$$= \int_0^1 \int_{\boxed{0}}^{\boxed{\sqrt{y}}} x - y \, dx \, dy$$

$$= \int_0^1 \left(\frac{1}{2}x^2 - xy \right)\Big|_0^{\sqrt{y}} dy$$

$$= \int_0^1 \frac{1}{2}(\sqrt{y})^2 - \sqrt{y} \cdot y \, dy = \int_0^1 \frac{1}{2}y - y^{3/2} \, dy$$

$$= \left(\frac{1}{4}y^2 - \frac{2}{5}y^{5/2}\right)\Big|_0^1 = \frac{1}{4} - \frac{2}{5} = \boxed{\frac{-3}{20}}$$



$$\iint x - y \, dy\,dx$$

2nd way!

$$\iint_W x - y \, dy\,dx$$

$$= \int_0^1 \boxed{\int_{x^2}^{1} x - y \, dy} \, dx$$

after this inside part, all the y's should we gone, only want x's in the bounds.

$$= \int_0^1 \left(xy - \frac{1}{2}y^2\right)\Big|_{x^2}^{1} dx$$

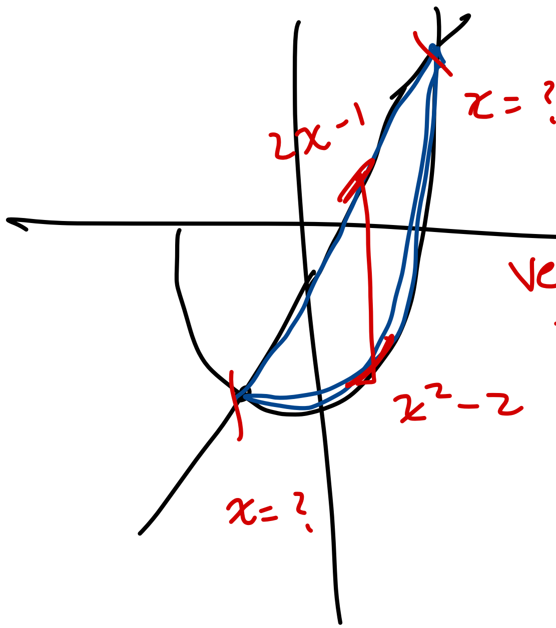
$$= \int_0^1 \left(x - \frac{1}{2}\right) - \left(x^3 - \frac{1}{2}x^4\right) dx$$

$$= \int_0^1 \frac{1}{2}x^4 - x^3 + x - \frac{1}{2}\, dx$$

$$= \frac{1}{10} - \frac{1}{4} + \frac{1}{2} - \frac{1}{2} = \boxed{\frac{-3}{20}}$$

3. Find the area of the region between the graphs of $y = 2x - 1$ and $y = x^2 - 2$.

$$\iint_W 1\, dA = \text{Area of } W$$



vertical $\Rightarrow$ $dy\,dx$

$$2x - 1 = x^2 - 2$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$\int_{1-\sqrt{2}}^{1+\sqrt{2}} \int_{x^2-2}^{2x-1} 1\, dy\, dx = \int_{1-\sqrt{2}}^{1+\sqrt{2}} 2x - 1 - (x^2 - 2)\, dx$$

$$= \int_{1-\sqrt{2}}^{1+\sqrt{2}} -x^2 + 2x + 1\, dx = \boxed{\frac{8\sqrt{2}}{3}} = \text{Area of region}$$

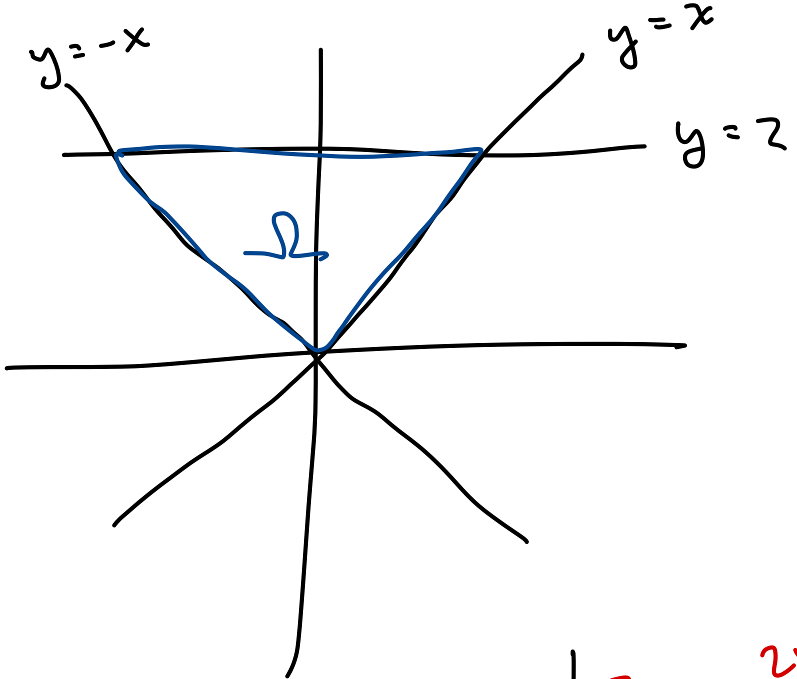
4. Integrate the function $f(x, y) = x + y^2 - 2$ on the region Ω bounded by the equations $y = 2$, $y = -x$ and $y = x$.

5. Evaluate the integral

$$\iint_D 2y \, dA$$

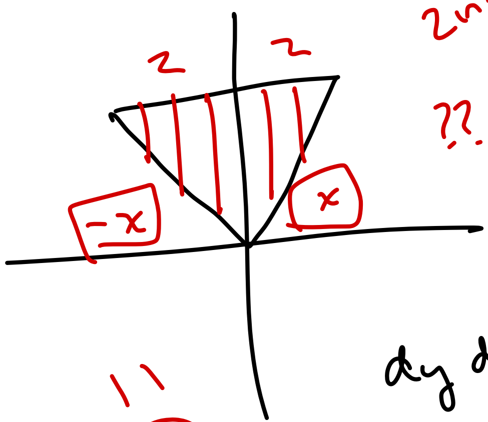
where D is the region bounded by the equations $y = e^{2x}$ and $y = (e^2 - 1)x + 1$.

4. Integrate the function $f(x, y) = x + y^2 - 2$ on the region Ω bounded by the equations $y = 2$, $y = -x$ and $y = x$.



Which order?

$$\iint_\Omega x + y^2 - 2 \, dx\, dy \quad \text{or} \quad \iint_\Omega x + y^2 - 2 \, dy\, dx$$

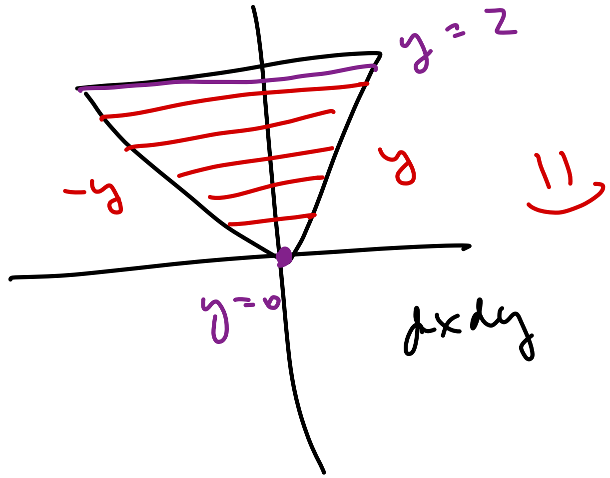


2 integrals required!

??

dy dx

$$\int_{-1}^{0}\int_{-x}^{2} + \int_{0}^{1}\int_{x}^{2} \quad \text{Bleh} \ldots$$



dx dy

nice!

$$\int_0^2 \int_{-y}^{y} x + y^2 - 2 \, dx\, dy$$

$$\int_0^2 \int_{-y}^{y} x + y^2 - 2 \, dx \, dy = \int_0^2 \left(\frac{1}{2}x^2 + xy^2 - 2x \right)_{-y}^{y} dy$$

$$= \int_0^2 \left(\cancel{\frac{1}{2}y^2} + y^3 - 2y \right) - \left(\cancel{\frac{1}{2}y^2} - y^3 + 2y \right) dy$$

$$= \int_0^2 2y^3 - 4y \, dy = \left(\frac{1}{2}y^4 - 2y^2 \right)_0^2$$

$$= \frac{1}{2}(2)^4 - 2(2)^2 = 8 - 8 = \boxed{0}$$

5. Evaluate the integral

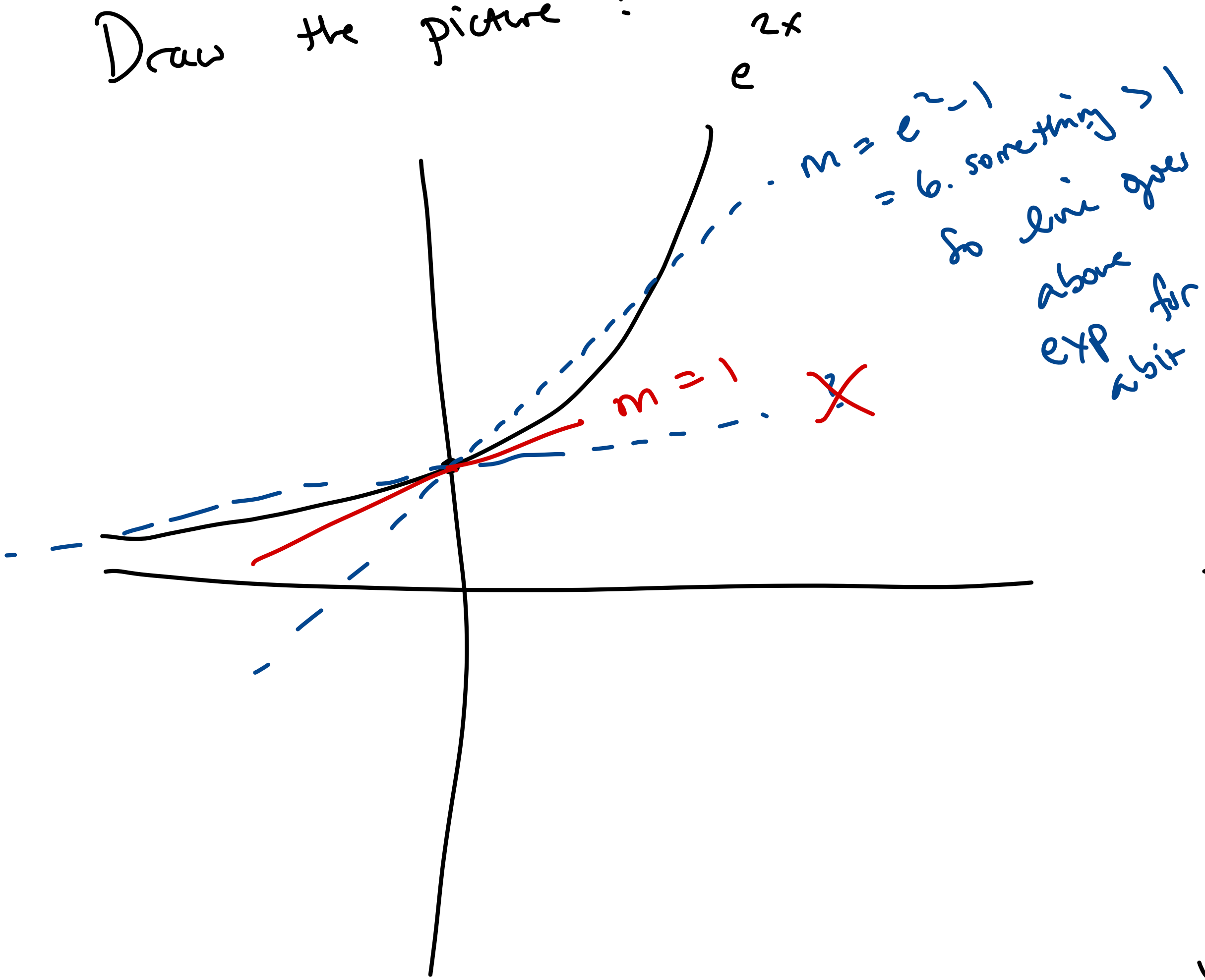
$$\iint_D 2y\,\boxed{dA}$$

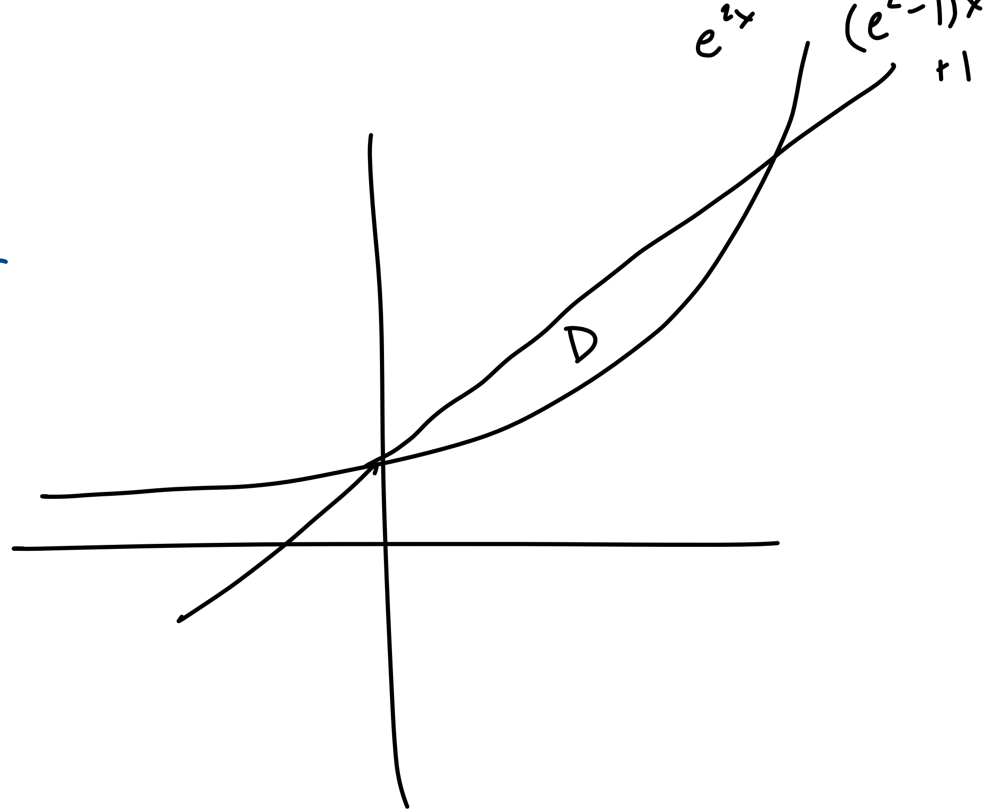
$m = e^2 - 1 = 6.\text{ something}$

where D is the region bounded by the equations $y = e^{2x}$ and $y = (e^2 - 1)x + 1$.

Draw the picture!

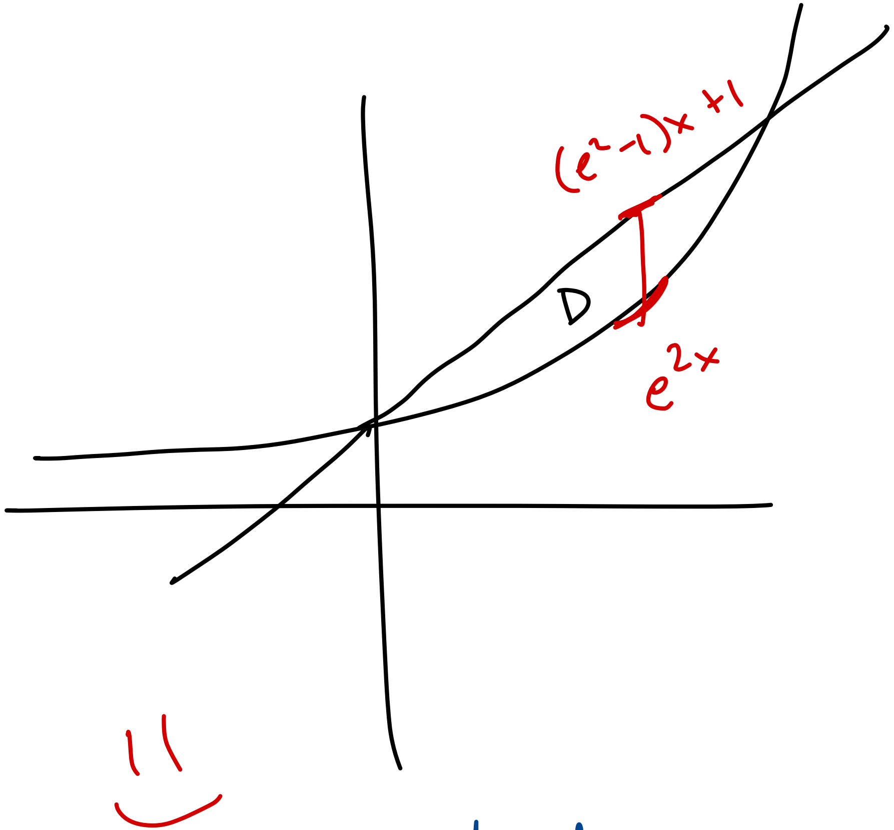
line!





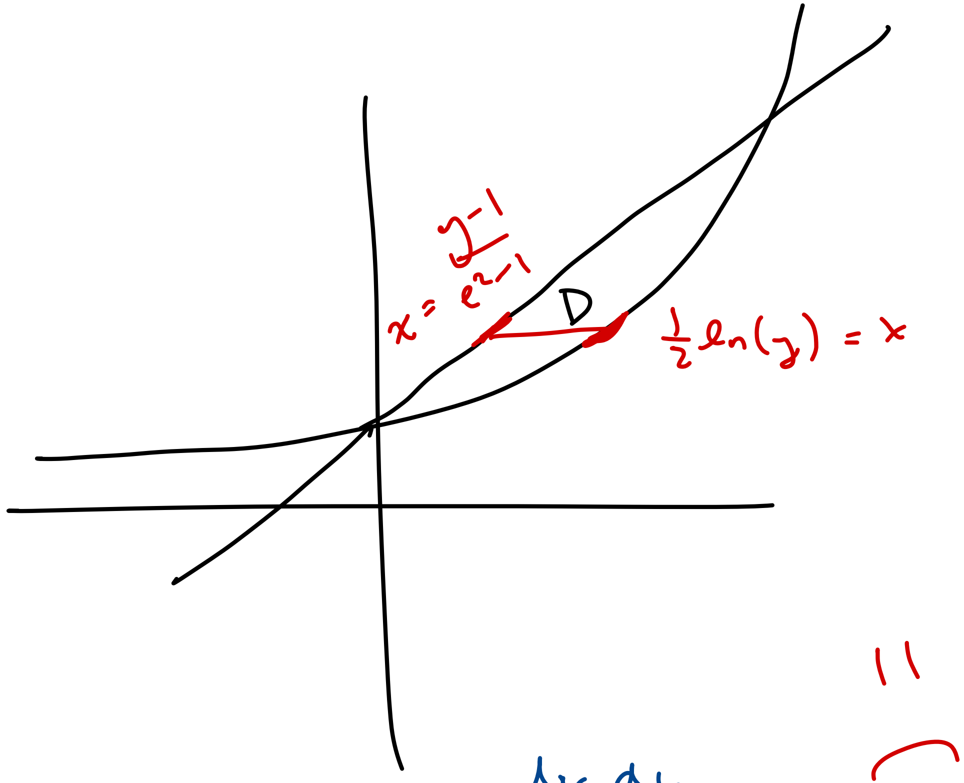
$$\iint_D 2y\,dy\,dx \quad \text{or} \quad \iint_D 2y\,dx\,dy$$

which one?



dy dx

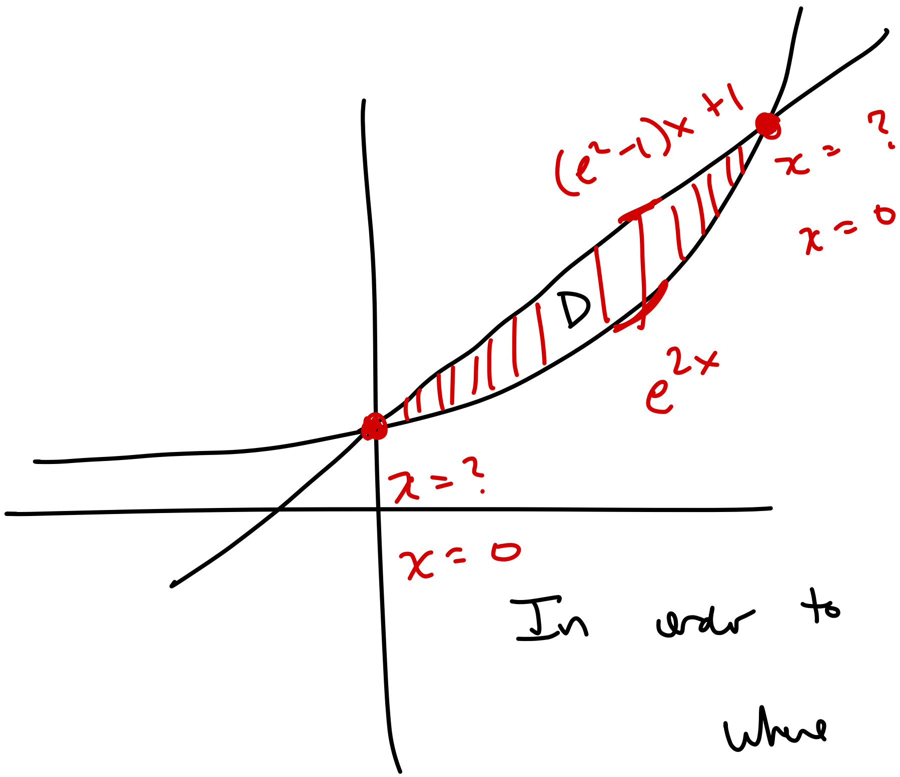
$$\int_{-}^{-}\int_{e^{2x}}^{(e^2-1)x+1} 2y \, dy \, dx$$



dx dy

$$\int \int_{\frac{y-1}{e^2-1}}^{\frac{1}{2}\ln(y)}$$

too annoying



$$\int_0^1 \int_{e^{2x}}^{(e^2-1)x+1} 2y \, dy \, dx$$

Integral to do!

In order to find the x bounds, we have to find where $y = e^{2x}$ intersects $y = (e^2-1)x+1$

$$e^{2x} = (e^2-1)x+1$$

has 2 solutions at $x=0$, $x=1$

$$e^{2x}-1 = (e^2-1)x$$

??? How???

In fact $x=0$, then $e^0 - 1 \stackrel{?}{=} (e^2-1)\cdot 0$

$1-1 = 0 = (e^2-1)\cdot 0$ ✓

$x=1$, then $(e^2-1) = (e^2-1)\cdot 1$ ✓

$$\int_0^1 \int_{e^{2x}}^{(e^2-1)x+1} 2y \, dy \, dx = \int_0^1 \left(y^2 \right)\Big|_{e^{2x}}^{(e^2-1)x+1} dx$$

$$= \int_0^1 (e^2-1)^2 x^2 + 2(e^2-1)x + 1 - e^{4x} \, dx$$

$$= \left(\frac{1}{3}(e^2-1)^2 x^3 + (e^2-1)x^2 + x - \frac{1}{4} e^{4x} \right)\Big|_0^1$$

$$= \left(\frac{1}{3}(e^2-1)^2 + (e^2-1) + 1 - \frac{1}{4} e^4 \right) - \left(-\frac{1}{4} \right)$$

$$= \boxed{\frac{1}{12} e^4 + \frac{1}{3} e^2 + \frac{19}{12}}$$

LAB 4

If $\vec{u}$ gives constant temp

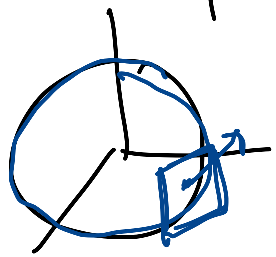
$$\Rightarrow \nabla_{\vec{u}} T = 0 \iff \nabla T \cdot \vec{u} = 0 \iff \overset{(9,1,2)}{\overline{\nabla T}} \perp \vec{u}$$

How to find $\vec{u}$ given ∇f?

Ex: $\nabla f = (1,2,3)$

$(1,2,3) \cdot \vec{u} = 0$ $\qquad$ $\vec{u} = (u_1, u_2, u_3)$

$T = c$ level set!



$u_1 + 2u_2 + 3u_3 = 0$

Pick a solution

$0 + 2u_2 + 3u_3 = 0$

$u_1 = 0 \quad u_2 = -3 \quad u_3 = 2$

Every direction vector of the tangent plane.

$0 + 2\cdot(-3) + 3\cdot(2) = 0$ ✓

$\vec{u} = (0,-3,2)$

$$\longrightarrow \frac{(0,-3,2)}{\|(0,-3,2)\|}$$

Midterm 1 24.

$$A = \begin{bmatrix} 2 & a & 0 \\ -1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} a & 1 \\ -2 & 1 \\ 3 & -3 \end{bmatrix}$$

Can any value of a make $AB = BA$?

$$AB = \begin{bmatrix} 2 & a & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 0 & a+2 \\ -a+1 & -3 \end{bmatrix}$$

$2 \times \boxed{3 \quad \checkmark \quad 3} \times 2$ $\qquad$ $\underline{2 \times 2}$

$$BA = \begin{bmatrix} a & 1 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} 2 & a & 0 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2a-1 & a^2+1 & 1 \\ -5 & -2a+1 & 1 \\ 9 & 3a-3 & -3 \end{bmatrix}$$

$3 \times \boxed{2 \quad \checkmark \quad 2} \times 3$ $\qquad$ $\underline{3 \times 3}$ $\qquad -1 + 1 - 3 = -3$

$$\int_0^1 \int_0^1 \ln((x+1)(y+1))\,dx\,dy$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\int_0^1 \int_0^1 \ln(x+1) + \ln(y+1)\,dx\,dy$$

$$\int \ln(x)\,dx = \int 1 \cdot \ln(x)\,dx = \int \underset{u}{\ln(x)}\ \underset{dv}{1\,dx}$$

$$v = x \qquad du = \frac{1}{x}\,dx$$

$$= uv - \int v\,du = x\ln(x) - \int x\frac{1}{x}\,dx$$

$$= x\ln(x) - \int 1\,dx = x\ln(x) - x$$

$$\cos(x) = 1 - x^2 + \frac{1}{4!}x^4 - \dots$$