General Stuff

- Office Hours

  T: 12:30 - 1:30, Th: 10 - 11

- Quiz 4 on Today (3/11)
- Topics include probably 5.5 and chapter 4 material. Probably 7.1 as well.

  1 problem

  15 minutes to take quiz

  5 minutes to upload to gradescope

  11:15 - 11:40 questions before quiz

  11:4~~0~~ 45 - 12:00 quiz

  12:00 - 12:05 uploading

- Lab after quiz today from 12:20 - 1:10

Arc length

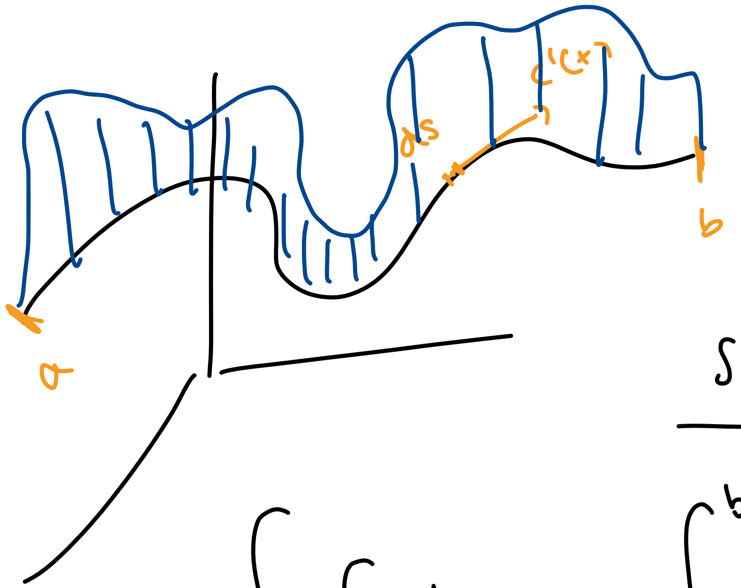
$c : [a,b] \longrightarrow \mathbb{R}^n$ $\qquad f : \mathbb{R}^n \to \mathbb{R}$

$$\int_c f \, ds = \underbrace{\int_a^b}_{c} \underbrace{f(c(t))}_{f} \underbrace{\|c'(t)\| \, dt}_{ds}$$

ds infinitesimal length along $c(t)$

$$\frac{ds}{dt} = \|c'(t)\|$$

speed vs velocity



Scalar line integral

$$\int_c f \, ds = \int_a^b f(c(t)) \, \|c'(t)\| \, dt$$

arc length

vector line integral

$$\int_c F \cdot d\vec{s} = \int F(c(t)) \cdot c'(t) \, dt$$

avg flow

Div, Grad, Curl Table

$$F(x,y,z) = (F_1, F_2, F_3)$$

Div	Grad	Curl
$\nabla \cdot F = \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$	$\nabla f = \left(\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}\right)$	$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \underline{F_1} & \underline{F_2} & \underline{F_3} \end{vmatrix}$
$\nabla \cdot$: vector field $\longrightarrow$ scalar function	∇ : scalar function $\longrightarrow$ vector field	$\nabla \times$: 3D vector field $\longrightarrow$ 3D vector field

Scalar functions $\xrightarrow{\text{Grad}}$ 3D vector fields $\xrightarrow{\text{Curl}}$ 3D vector fields $\xrightarrow{\text{Div}}$ Scalar functions

ϕ $\qquad$ $\nabla \phi$, F $\qquad$ $\nabla \times F$, G $\qquad$ $\nabla \cdot G$

Doing 2 arrows in a row always gives you 0!

- $\nabla \times (\nabla \phi) = 0$ always!
- $\nabla \cdot (\nabla \times F) = 0$ always!

Let $f(x,y,z) = xe^y + z\sin(x)$. Find ∇f.

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= \left(\frac{\partial}{\partial x}\left(xe^y + z\sin(x)\right), \frac{\partial}{\partial y}\left(\ldots\right), \frac{\partial}{\partial z}\left(\ldots\right) \right)$$

$$= \left(e^y + z\cos(x),\ xe^y,\ \sin(x)\right)$$

∇f is a vector field!

1. Let $F(x, y, z) = (xz, e^y, x+y+z)$. (a) Which of the following are well-defined, $\nabla\cdot(\nabla\times F)$ or $\nabla\times\nabla F$. (b) Find $\nabla\times F$ and $\nabla\cdot F$.

which make sense?

(a) $\nabla\cdot(\nabla\times F)$ makes sense! In fact it's always 0.

$\nabla\times\nabla F$ makes no sense because you can't take ∇F, F is not a scalar function

(b)
$$\nabla\times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & e^y & x+y+z \end{vmatrix} = \hat{\imath}\left(\overset{1}{\frac{\partial}{\partial y}(x+y+z)} - \overset{0}{\frac{\partial}{\partial z}(e^y)}\right) - \hat{\jmath}\left(\overset{1}{\frac{\partial}{\partial x}(x+y+z)} - \overset{x}{\frac{\partial}{\partial z}(xz)}\right) + \hat{k}\left(\overset{0}{\frac{\partial}{\partial x}(e^y)} - \overset{0}{\frac{\partial}{\partial y}(xz)}\right)$$

$$= (\underline{1}, x-1, 0).$$

$$\nabla\cdot F = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(e^y) + \frac{\partial}{\partial z}(x+y+z)$$

$$= z + e^y + 1$$

2. Let $F(x, y, z) = (2xy+z\cos(x), x^2, \sin(x))$. Compute out the curl and show that $\nabla\times F = 0$. Explain why F has a potential function $\phi(x, y, z)$.

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+z\cos x & x^2 & \sin(x) \end{vmatrix}$$

$$= \hat{i}\left(\overset{0}{\cancel{\frac{\partial}{\partial y}(\sin(x))}} - \overset{0}{\cancel{\frac{\partial}{\partial z}(x^2)}} \right) - \hat{j}\left(\overset{\cos x}{\frac{\partial}{\partial x}(\sin(x))} - \overset{\cos x}{\frac{\partial}{\partial z}(2xy+z\cos x)} \right)$$

$$+ \hat{k}\left(\overset{2x}{\frac{\partial}{\partial x}(x^2)} - \overset{2x}{\frac{\partial}{\partial y}(2xy + z\cos x)} \right)$$

$$= (0-0, -\cos x + \cos x, 2x - 2x) = (0,0,0)$$

2. Let $F(x, y, z) = (2xy + z\cos(x), x^2, \sin(x))$. Compute out the curl and show that $\nabla \times F = 0$. Explain why F has a potential function $\phi(x, y, z)$.

Fact: If a vector field is defined everywhere and $\nabla \times F = 0$, then F has a potential!

I.E. scalar function ϕ such that

$$\nabla \phi = F.$$

Yes, it is the gradient of a potential since F is defined everywhere and

$$\nabla \times F = 0.$$

Related: If $\nabla \times F \neq 0$, there's never a potential function!