General Stuff

- Office Hours

  T: 12:30 - 1:30, Th: 10 - 11

- Lab 07 due tonight.

  Let me know if Segments and Estimate don't work for you

- Midterm 4 on Thursday (3/18)

- Topics include ~~probably 5.5 and chapter 4 material. Probably 7.1 as well.~~ 7.1, 7.2

  2 problems

  30 minutes to take exam

  5-10 minutes to upload to gradescope

  11:15 - 11:25 questions before quiz

  11:25 - 11:55 quiz

  11:55 - 12:05 uploading

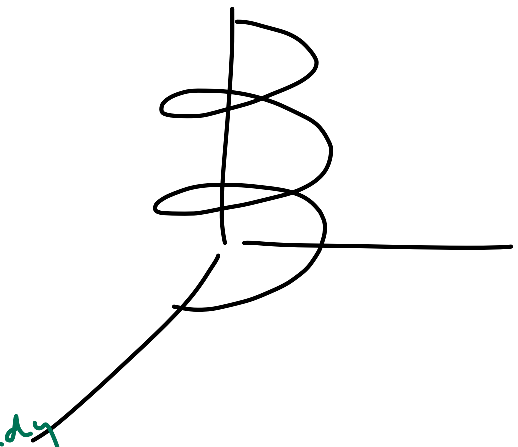
Scalar Line Integral vs Vector Line Integral

Scalar	arclength or total mass of a wire Integral of a scalar function	$\int_c f\, ds$ "speed" $= \int_a^b f(c(t))\, \lVert c'(t) \rVert\, dt$
vector	work done by vector field (force field) Integral of a vector field	$\int_c F \cdot d\vec{s}$ "velocity" $= \int_a^b F(c(t)) \cdot c'(t)\, dt$

1. Calculate the integral

$$\int_C \cos(z)\,dx + \sin(z)\,dy + (x+y)\,dz$$??

where C is the helix $c(t) = (-\sin(t), \cos(t), t)$. $t = 0 \to t = 2\pi$



$$\int_C \cos(z)\,dx + \sin(z)\,dy + (x+y)\,dz$$

This is the dot product $F \cdot d\vec{s}$ already written out!

$$= \int_C (\cos(z), \sin(z), x+y) \cdot \underbrace{(dx, dy, dz)}_{}$$

$d\vec{s}$ ''

$$= \int_C (\cos(z), \sin(z), x+y) \cdot d\vec{s}$$

(vector line integral!)

$$\int_c (\cos(z),\ \sin(z),\ x+y) \cdot d\vec{s} \qquad c(t) = (-\sin(t),\ \cos(t),\ t) \qquad \begin{matrix} t=0 \\ t=2\pi \end{matrix}$$

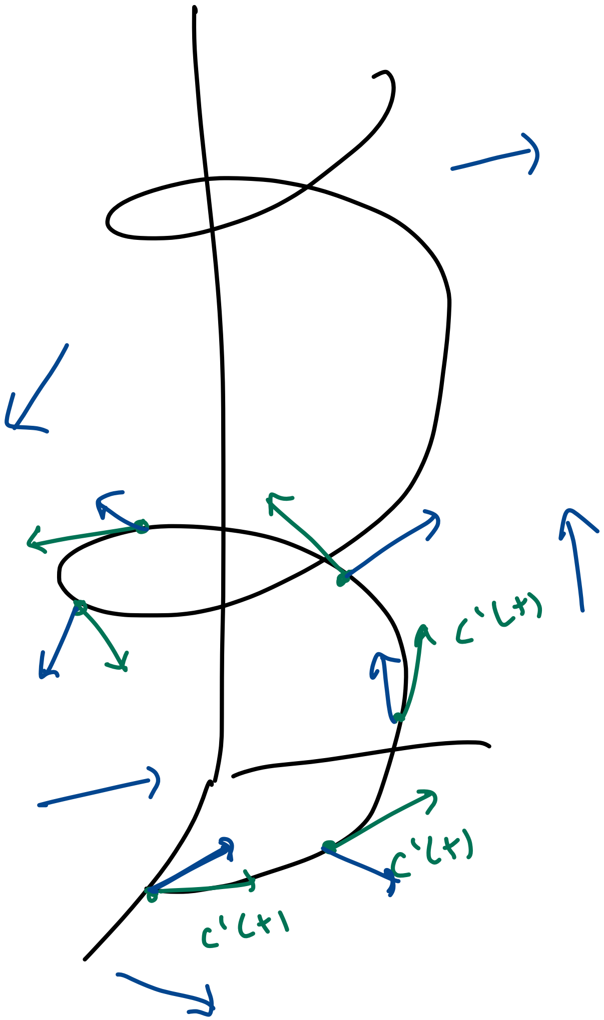
$$x \qquad y \qquad z = t$$

$$\int_0^{2\pi} F(c(t)) \cdot c'(t)\ dt = \int_0^{2\pi} (\cos(t),\ \sin(t),\ -\sin(t)+\cos(t)) \cdot (-\cos(t),\ -\sin(t),\ 1)\ dt$$

$$= \int_0^{2\pi} \underbrace{-\cos^2(t) \cancel{+} -\sin^2(t)}_{-1} - \sin(t) + \cos(t)\ dt$$

$$= \int_0^{2\pi} -1 - \sin(t) + \cos(t)\ dt = \Big(-t + \cos(t) + \sin(t) \Big)_0^{2\pi}$$

$$= -2\pi$$



$F(x,y,z) = (-\sin(z), \cos(z), x+y)$

$\int_c F \cdot ds$ = total work done by the particle as the particle moves

$F \cdot c'(t) = \|F\| \, \|c'(t)\| \cos\theta$

$W = F \cdot \Delta x$

$\Rightarrow W = \int F \cdot d\vec{s}$

$$\int_C \cos(z)\,\underset{\frac{dx}{dt}\,dt}{\underset{\downarrow}{dx}} + \sin(z)\,\underset{\frac{dy}{dt}\,dt}{\underset{\downarrow}{dy}} + (x+y)\,\underset{\frac{dz}{dt}\,dt}{\underset{\downarrow}{dz}}$$

$$c(t) = (\overset{x}{-\sin(t)}, \overset{y}{\cos(t)}, \overset{z}{t})$$

$$0 \to 2\pi$$

$$\int_0^{2\pi} \cos(t)\left(\frac{d}{dt}(-\sin(t))\right)dt + \sin(t)\left(\frac{d}{dt}(\cos(t))\right)dt + (-\sin(t)+\cos(t))\cdot\left(\frac{d}{dt}(t)\right)dt$$

$$= \int_0^{2\pi} \cos(t)\cdot(-\cos(t)) + \sin(t)\cdot(-\sin(t)) + (-\sin(t)+\cos(t))\,dt$$

$$= \int_0^{2\pi} -\cos^2(t) \cancel{-} \sin^2(t) - \sin(t) + \cos(t)\,dt = 2\pi$$

2. Suppose a wire can be parametrized as the intersection of the plane $z = y+2$ and $x^2+y^2 = 4$. Suppose the mass density function is given by $m(x, y, z) = z(x^2 + y^2 + 1)$. ~~Find the total~~ Set up the integral. mass of the wire.

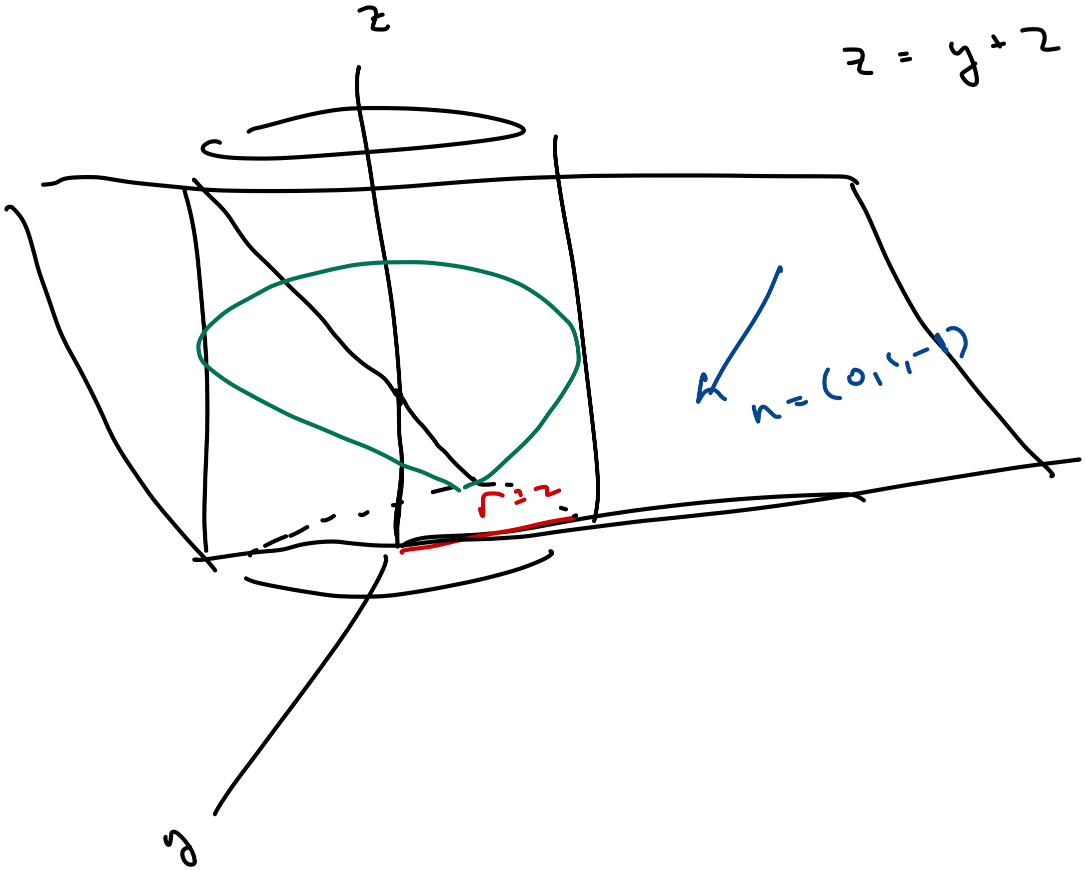
vector

scalar

so it's a scalar function!

$$M = \int_{C} m \, ds$$

scalar line integral



$$y - z = -2$$

$$n = (0, 1, -1)$$

$$c(t) = (\overbrace{2\cos(t), 2\sin(t)}^{x^2+y^2=4}, \underbrace{2\sin(t) + 2}_{z = y+2})$$

$$t = 0 \quad t = 2\pi$$

$$\boxed{\begin{array}{l} x^2 + z^2 = 4 \\ y = z + 2 \\ (2\cos(t), 2\sin(t) + 2, 2\sin(t)) \end{array}}$$

$$M = \int_C z\,(x^2+y^2+1)\,ds = \int_0^{2\pi} m(c(t))\,\underbrace{\|c'(t)\|}\,dt$$

$$c(t) = (\underset{x}{2\cos(t)},\ \underset{y}{2\sin(t)},\ \underset{z}{2\sin(t)+2})$$

$$c'(t) = (-2\sin(t),\ 2\cos(t),\ 2\cos(t))$$

$$\underbrace{\|c'(t)\|} = \sqrt{\underbrace{\cancel{4\sin^2(t) + 4\cos^2(t)}}_{4} + 4\cos^2(t)}$$

$$= \sqrt{4 + 4\cos^2(t)} = \underbrace{2\sqrt{1+\cos^2(t)}}$$

$$= \int_0^{2\pi} (2\sin(t)+2)\left((2\cos(t))^2 + (2\sin(t))^2 + 1\right) 2\sqrt{1+\cos^2(t)}\,dt$$

$$= \int_0^{2\pi} \left(2\sin(t) + 2\right)\left(2+1\right) 2\sqrt{1+\cos^2(t)} \, dt$$

$$\underbrace{= 6\int_0^{2\pi} \left(2\sin(t) + 2\right)\sqrt{1+\cos^2(t)} \, dt}$$ ✓ $= 91.6847\ldots$

$$= -12\int_0^{2\pi} -\sin(t)\sqrt{1+\cos^2(t)} \, dt + 12\int_0^{2\pi} \sqrt{1+\cos^2(t)} \, dt$$

$u = \cos(x)$

$du = -\sin(t)\,dt$

$$-12\int \sqrt{1+u^2} \, du \qquad \text{etc}$$

3. Suppose a force field on $\mathbb{R}^2$ is given by $F(x,y) = (x+y, x^2-y)$. Find the work done by F on a particle moving along the trajectory given by $p(t) = (t, t^2-t+1)$. $t=0, t=3$

$$W = \int_c F \cdot d\vec{s} = \int_0^3 F(p(t))\ p'(t)\,dt \qquad p'(t) = (1, 2t-1)$$

$$= \int_0^3 (t + t^2 - t + 1,\ t^2 - t^2 + t - 1) \cdot (1, 2t-1)\,dt$$

$$= \int_0^3 (t^2+1,\ t-1) \cdot (1, 2t-1)\,dt$$

$$= \int_0^3 (t^2+1) + (t-1)(2t-1)\,dt = \int_0^3 3t^2 - 3t + 2\,dt$$

$$= 39/2$$

4. Consider the circle of radius 1 given by $c(t) = (\cos(t), \sin(t))$. Consider the parametrization of $d(t) = (\sin(t), \cos(t))$. Determine some new bounds for d which agree with c, and determine whether d is orientation preserving or reversing.

$$\int_c F \cdot d\vec{s}$$

$c_1(t)$ vs $c_2(t)$ are 2 different parametrizations

$$\int F(c_1(t)) \cdot c_1'(t)\,dt = \pm \int F(c_2(t)) \cdot c_2'(t)\,dt$$

The minus occurs because c_2 might parametrize the curve backwards from c_1.

c_2 is either orientation preserving

$\longrightarrow c_1$

$\longrightarrow c_2$

same direction

c_2 is orientation reversing

$\longrightarrow c_1$

$\longleftarrow c_2$

$c(t) = (\cos(t), \sin(t))$ counter clockwise starting at $\theta = 0$

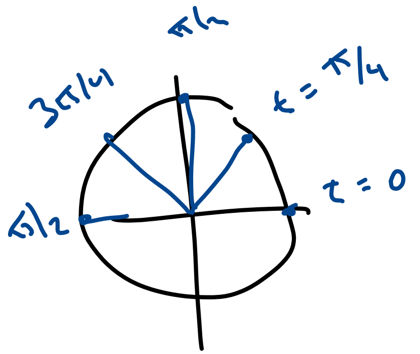
$d(0) = (0,1)$ $\qquad d(\pi/2) = (1,0)$

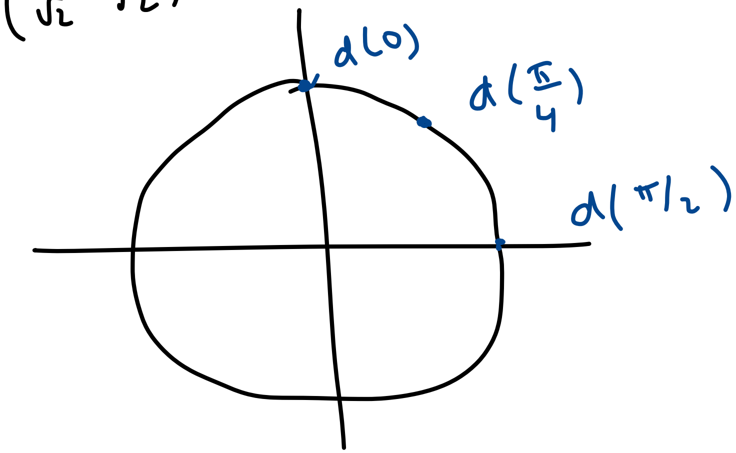
$d(t) = (\sin(t), \cos(t))$

$d(\pi/4) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Clockwise

1





d is orientation reversing!

$$\int F(c(t)) \cdot c'(t)\, dt = -\int F(d(t)) \cdot d'(t)\, dt$$

5. Find the arclength of the graph $y = \ln(\sec(x))$ from $x = 0$ to $x = \cancel{\pi}$. $\pi/4$

$$\text{Arclength} = \int_c 1 \, ds = \int_a^b \|c'(t)\| \, dt$$

$$c = (t, \ln(\sec(t))) \qquad c'(t) = \left(1, \frac{1}{\sec(t)} \sec(t)\tan(t)\right)$$

$$= (1, \tan(t))$$

$$\|c'(t)\| = \sqrt{1 + (\tan(t))^2} = \sqrt{1 + \tan^2(t)} = \sqrt{\sec^2(t)} = \sec(t)$$

$$\text{Arclength} = \int_c 1 \, ds = \int_0^{\pi/4} \|c'(t)\| \, dt = \int_0^{\pi/4} \sec(t) \, dt$$

$$= \Big(\ln(\tan(t) + \sec(t))\Big)\Big|_0^{\pi/4} = \ln(1 + \sqrt{2}) - \ln(1)$$

$$= \ln(1 + \sqrt{2}) = 0.88\ldots$$