General Stuff

- Office Hours

  T: 12:30 - 1:30, Th: 10 - 11

  Lab 05 due by the end of today
  Part 1e is optional
  a-d required

- Midterm 3 3/4 (best day for a parade, March 4th)

  2 problems

  30 minutes to take exam

  5-10 minutes to upload to gradescope

  11:15 - 11:25 questions before quiz

  11:25 - 11:55 quiz

  11:55 - 12:05 uploading

  Midterm material
  5 4 - 5.5 (?)
  Update coming (?)

- Lab after quiz from 12:20 - 1:10

1. Change the order of integration for the integral

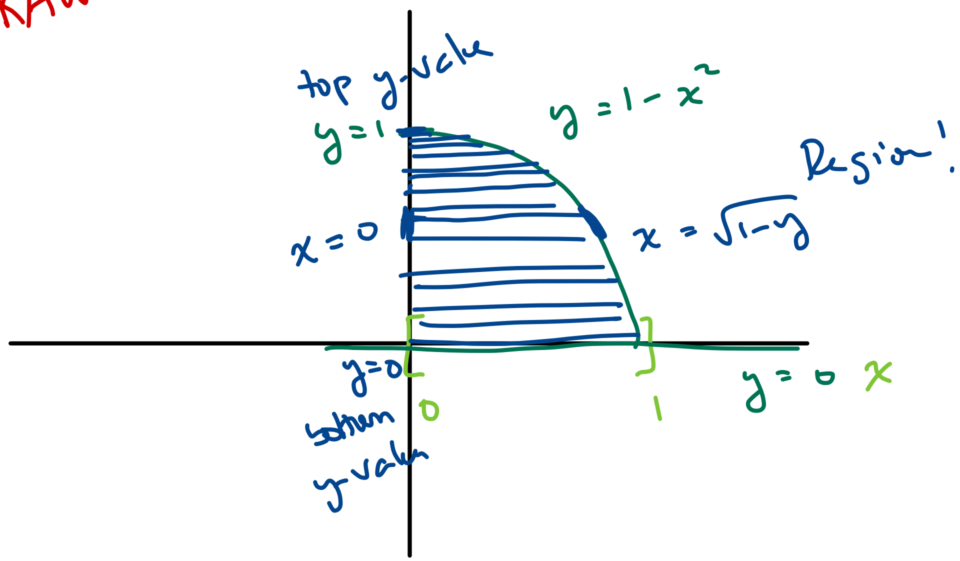
$$\int_0^1 \int_0^{1-x^2} 2x\,dy\,dx. \qquad = \int_{-}^{-}\int_{-}^{-} 2x\,\underbrace{dx\,dy}$$

$y = 1 - x^2$

$y = 0$

different bounds!

DRAW THE PICTURE!



$$y = 1 - x^2$$
$$x^2 = 1 - y$$
$$x = \pm\sqrt{1-y}$$
$$x = +\sqrt{1-y}$$

If dx is first, the first bounds should be functions of y. After dx integral all the x's should be gone.

$$\int_a^b \int_{f(y)}^{g(y)} 2x\,dx\,dy = \int_0^1 \int_0^{\sqrt{1-y}} 2x\,dx\,dy$$

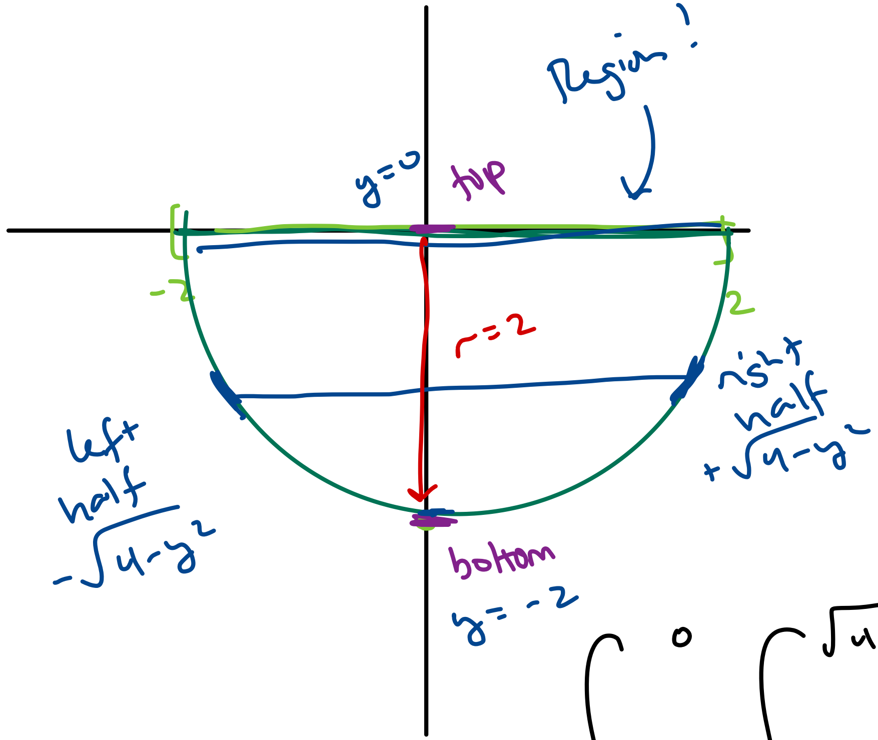
2. Change the order of integration

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{0} 1\,dy\,dx. \quad = \text{Area of region}$$

$y=0$

$$\int_a^b \int_{f(y)}^{g(y)} 1 \; dx\,dy$$

Draw the picture.



$$y = -\sqrt{4-x^2}$$

bottom half

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4 \implies$$ circle of radius 2 centered at origin

$$x = \pm\sqrt{4-y^2}$$

$$x = -\sqrt{4-y^2}$$ left half

$$x = +\sqrt{4-y^2}$$ right half

$$\int_{-2}^{0}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 1 \, dx\,dy$$

3. Bound the integral

$$\int_0^2 \int_0^3 \frac{1}{1+x^2y^2}\, \cancel{dA}\ dx\, dy$$

using the Mean Value Inequality.

$dy\, dx$ $\frac{1}{\text{Area}(\Omega)} \iint_\Omega f(x,y)\,dA = f(x_0,y_0)$ for some $(x_0,y_0) \in \Omega$.

average value

Thm If on Ω $\underset{\text{min}}{m} \le f(x,y) \le \underset{\text{max}}{M}$

$$\Longrightarrow m \cdot \text{Area}(\Omega) \le \iint_\Omega f(x,y)\,dA \le M \cdot \text{Area}(\Omega)$$

What's the region?

$\boxed{\Omega = [0,3] \times [0,2]}$ $\text{Area}(\Omega) = 3 \cdot 2 = 6$

min / max ?

$f(x,y) = \frac{1}{1+x^2y^2}$ this fraction has the largest value when the * denominator is smallest! when $x=y=0$

$$M = f(0,0) = \frac{1}{1} = 1.$$

This fraction is smallest when x, y are as big as possible!

* Large denom $\Rightarrow$ small fraction

$$m = f(3,2) = \frac{1}{1+3^2\cdot 2^2} = \frac{1}{37}$$

$$m \cdot \text{Area}(\Omega) \leq \iint_\Omega f(x,y)\,dA \leq M \cdot \text{Area}(\Omega)$$

$$\frac{1}{37} \cdot 6 \leq \int_0^2 \int_0^3 f(x,y)\,dx\,dy \leq 1 \cdot 6$$

$$\frac{6}{37} \leq \int_0^2 \int_0^3 \frac{1}{1+x^2y^2}\,dx\,dy \leq 6$$

$$\iint_\Omega e^{-x^2-y^2}\,dA$$

don't evaluate!

4. Set up the triple integral (!)

$$\iiint_{\Omega} 2z\, dV$$

where Ω is the region bounded by $x = 2 - y^2 - z^2$ and $x = z$.

Ω - 3D region!

$\subseteq \mathbb{R}^3$

↑ subset

$dV = \begin{cases} \boxed{dx}\, dy\, dz \\ \boxed{dx}\, dz\, dy \\ \boxed{dy}\, dx\, dz \\ \boxed{dy}\, dz\, dx \\ \boxed{dz}\, dy\, dx \\ \boxed{dz}\, dx\, dy \end{cases}$

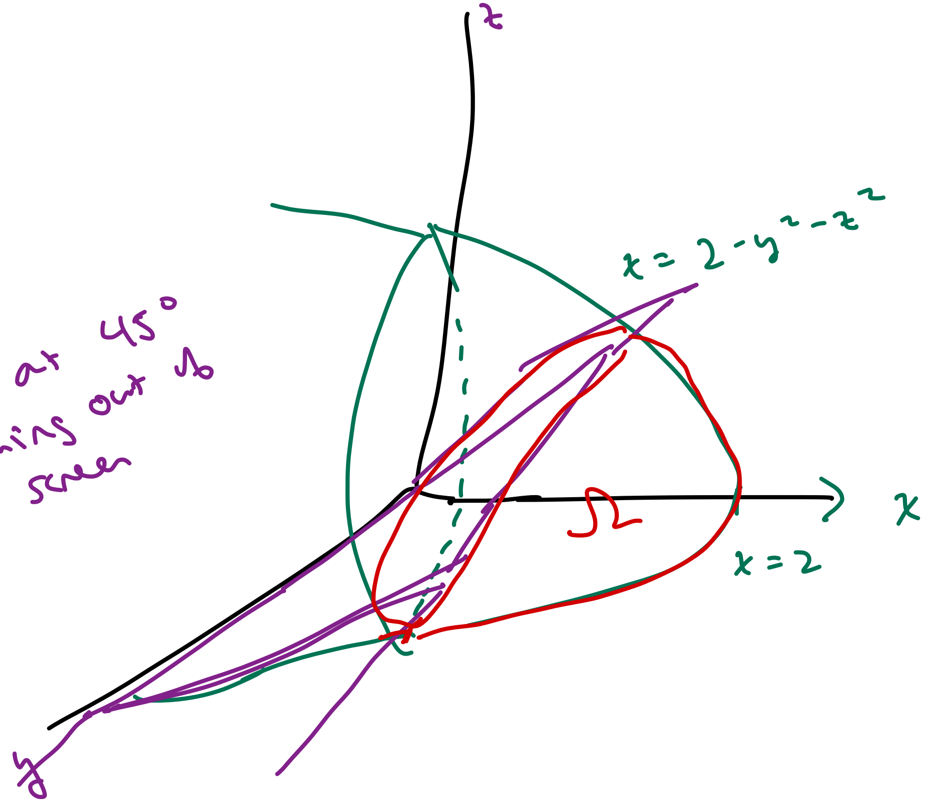
Which one is best?

$x = f(y, z)$

$x = 2 - y^2 - z^2$ ✓

" $z = 2 - y^2 - x^2$ " ✗

Draw the picture!



$x = z$

$x - z = 0$

$n = (1, 0, -1)$

$a = (0, 0, 0)$

$$\iiint_{\Omega} 2z \, dV$$

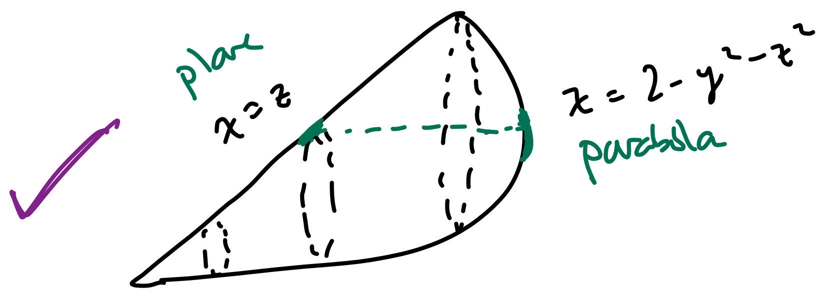
$$\iint \int_{f(y,z)}^{g(y,z)} 2z \, dx \, dA$$

$(x = 2 - y^2 - z^2)$

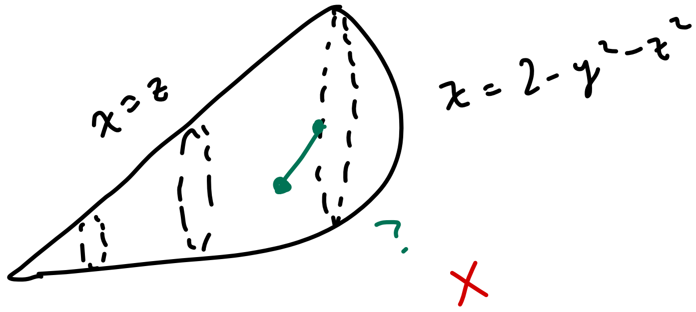
$$= \iint \int_{z}^{2-y^2-z^2} 2z \, dx \, dA$$

$(x = z)$

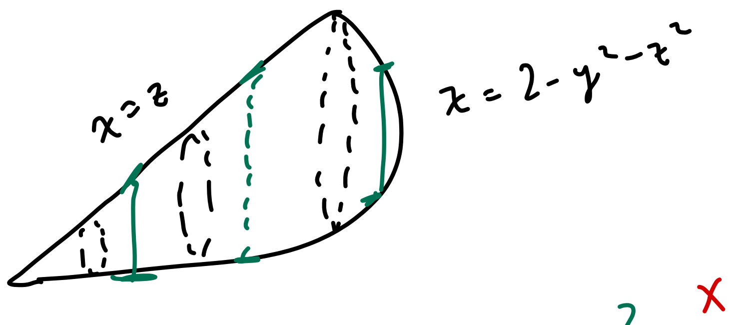
For the $dy\,dz$ or $dz\,dy$ part project Ω onto yz-plane



dx first

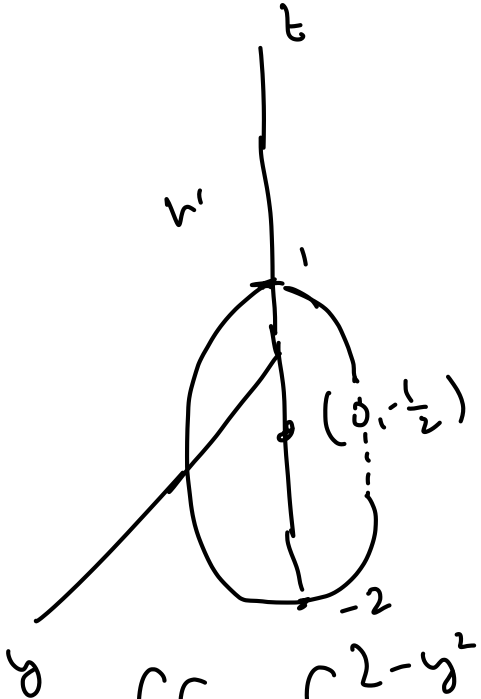


dy first

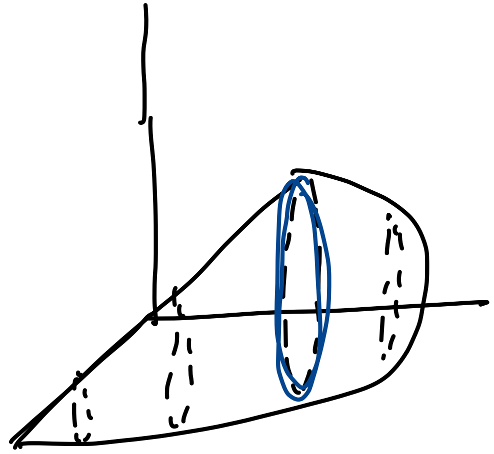


dz first, requires 2 integrals

Which of these makes bounds as easy as possible?



shadow



The shadow comes from when $x = 2 - y^2 - z^2$ and $x = z$ intersect!

$$\iint_W \int_z^{2-y^2-z^2} 2z \, dx \, dA$$

$$z = 2 - y^2 - z^2$$

$$y^2 + z^2 + z = 2$$ complete square

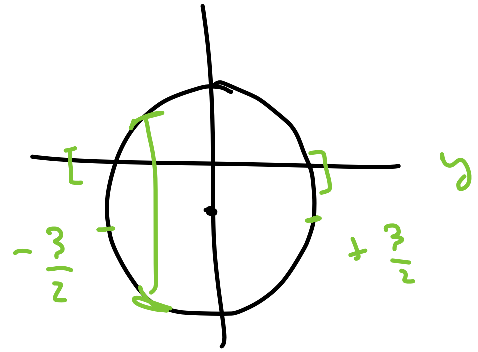
$$y^2 + z^2 + z + \frac{1}{4} = \frac{9}{4}$$

$$y^2 + \left(z + \frac{1}{2}\right)^2 = \frac{9}{4}$$

Circle of radius $\frac{3}{2}$ centered at $(0, -\frac{1}{2})$.

So the W integral is

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} \int_{-\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}}^{+\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}}$$



$$\Longrightarrow \int_{-\frac{3}{2}}^{\frac{3}{2}} \int_{-\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}}^{\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}} \int_{z}^{\frac{9}{2}-y^2-z^2} 2z \; dx \; dy \; dz$$