General Stuff

- Office Hours

  T: 12:30 - 1:30, Th: 10 - 11

  Office hours after class today , Lab 06 due tonight

- Quiz 4 on 3/11
- Topics include probably 5.5 and chapter 4 material. Probably 7.1 as well.

  1 problems

  15 minutes to take quiz

  5 minutes to upload to gradescope

  11:15 - 11:40 questions before quiz

  11:40 - 12:00 quiz

  12:00 - 12:05 uploading

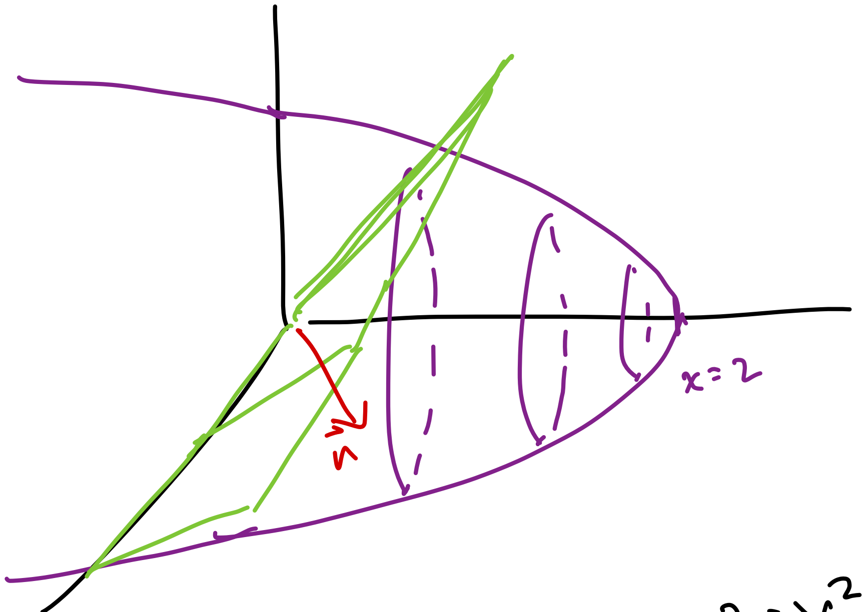
- Lab after quiz Thursday from 12:20 - 1:10

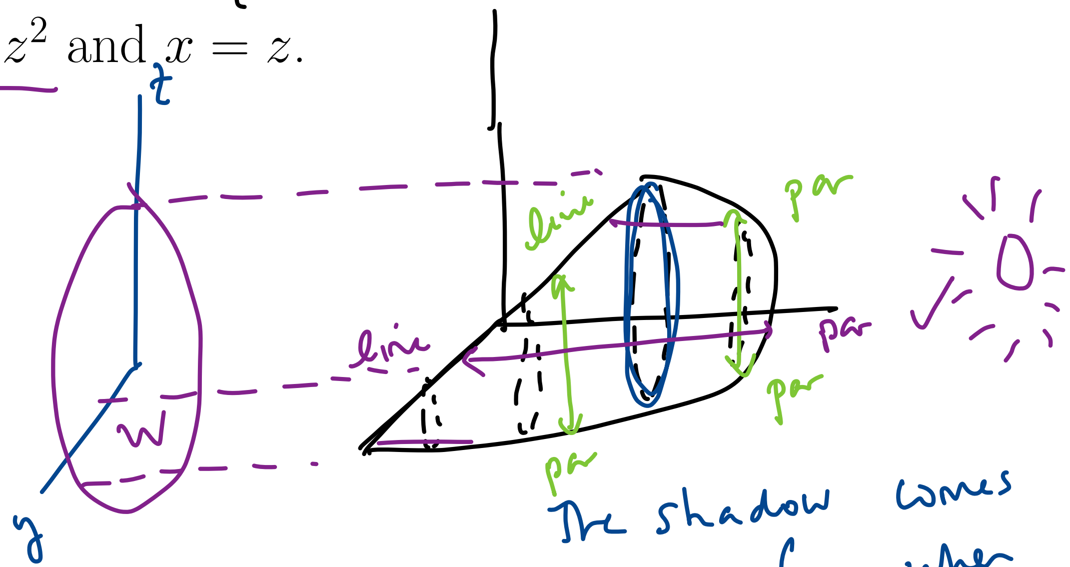
1. Set up the triple integral (!)

$$\iiint_\Omega 2z\,dV$$

where Ω is the region bounded by $x = 2 - y^2 - z^2$ and $x = z$.

$x - z = 0$, $n = (1, 0, -1)$





The shadow comes from when $x = 2 - y^2 - z^2$ and $x = z$ intersect!

$dy\,dz$, $dz\,dy$

$$\iint_W \int_z^{2-y^2-z^2} 2z\,dx\,dA$$

x-simple

$x = 2 - y^2 - z^2$

$x = z$

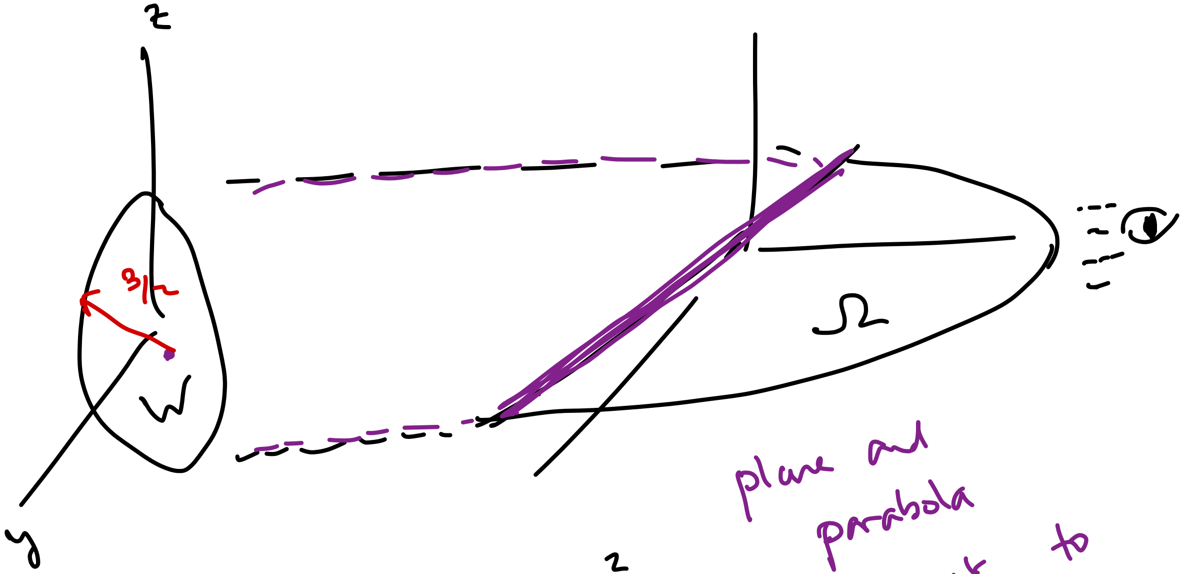
$\Rightarrow \quad z = 2 - y^2 - z^2$

$y^2 + z^2 + \boxed{z} = 2$

not centred at the origin!

$y^2 + \underbrace{z^2 + 2\left(\tfrac{1}{2}z\right) + \tfrac{1}{4}} = \frac{9}{4}$ - Complete the square

$y^2 + \left(z + \tfrac{1}{2}\right)^2 = \frac{9}{4}$

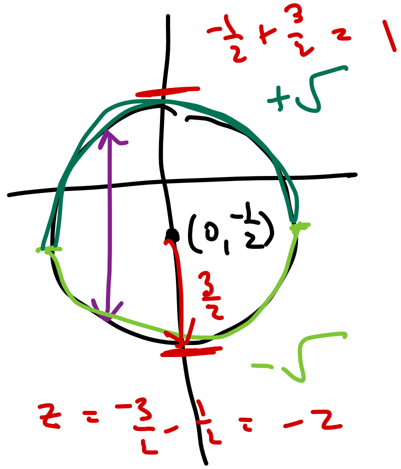


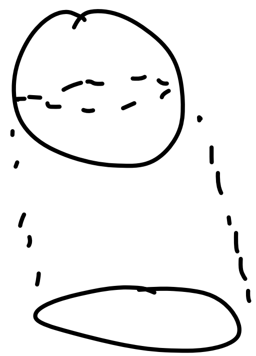
plane and parabola intersect to make the shadow!

$(x + y)^2 = x^2 + 2xy + y^2$

$\quad\; z \;\; \tfrac{1}{2} \qquad\quad z \;\; 2\left(\tfrac{1}{2}\right)z + \tfrac{1}{4}$

W is a circle centred $\left(0, -\tfrac{1}{2}\right)$, rad is $\sqrt{\frac{9}{4}} = \frac{3}{2}$.





$y = \pm\sqrt{\frac{9}{4} - \left(z + \tfrac{1}{2}\right)^2}$

*

$$\int_{-2}^{1}\int_{-\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}}^{+\sqrt{\frac{9}{4}-(z+\frac{1}{2})^2}}\int_{z}^{2-y^2-z^2} 2z \; dx \; dy \; dz$$

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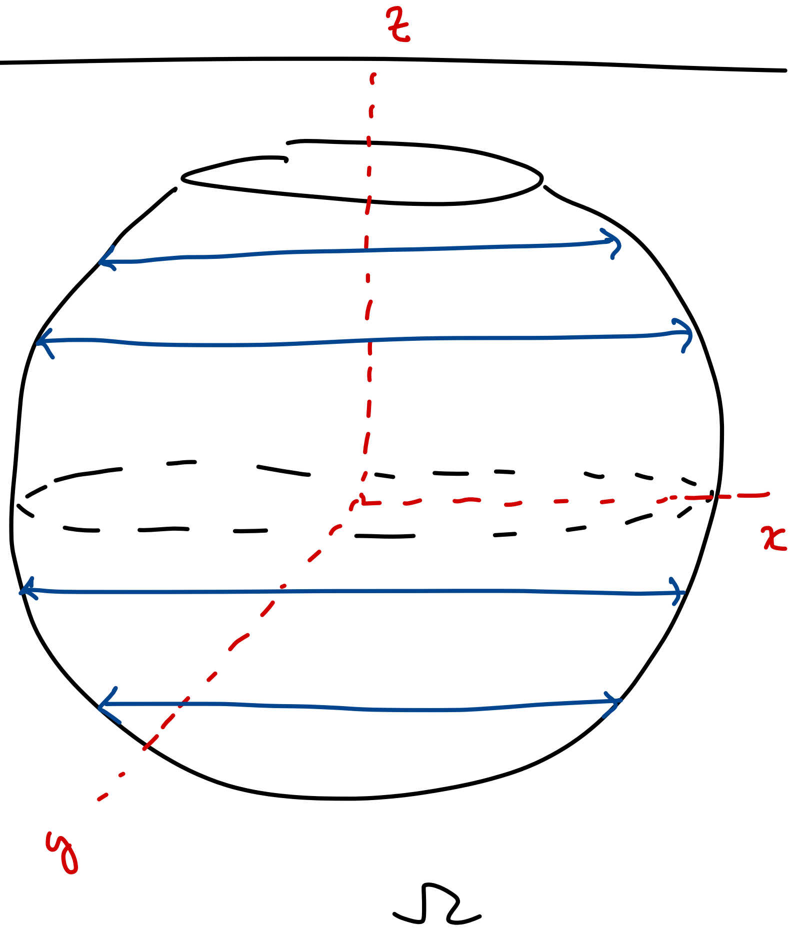
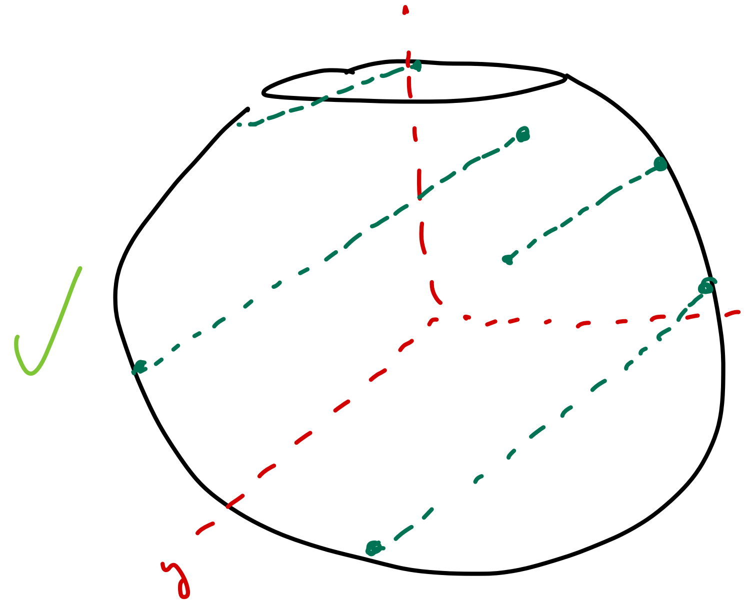
x - simple - draw a horizontal line. are the boundary functions always the same?

Since, the x - lines always go from left half to right half, this region is x - simple



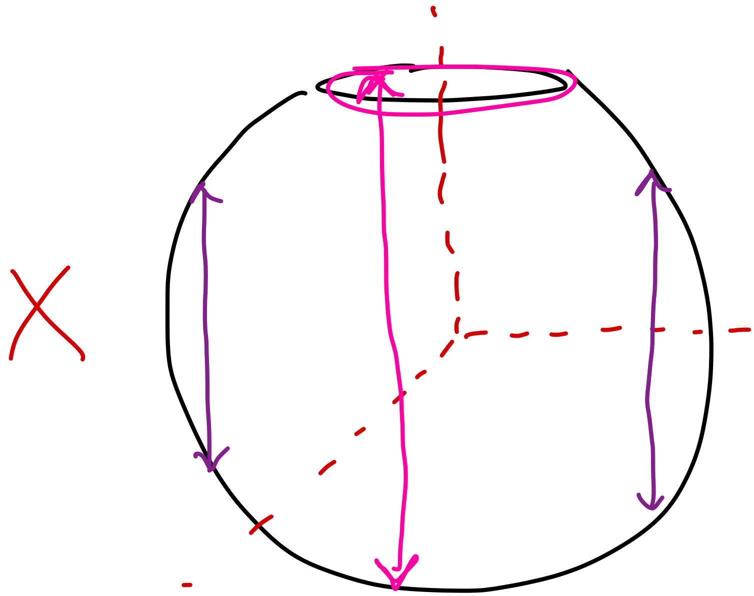
y - simple

The y-lines always go from the front half to back half, so it's y simple also.

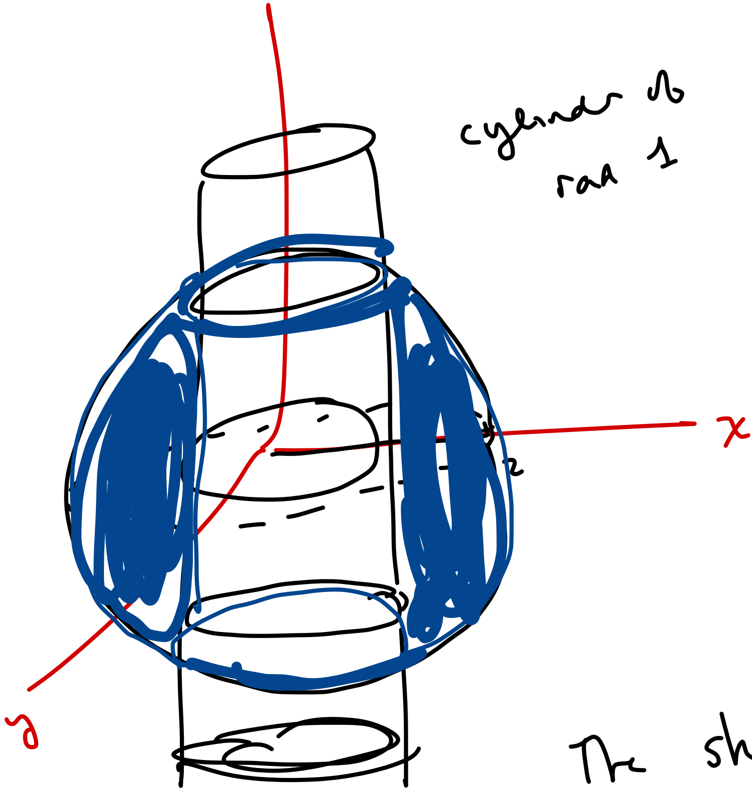


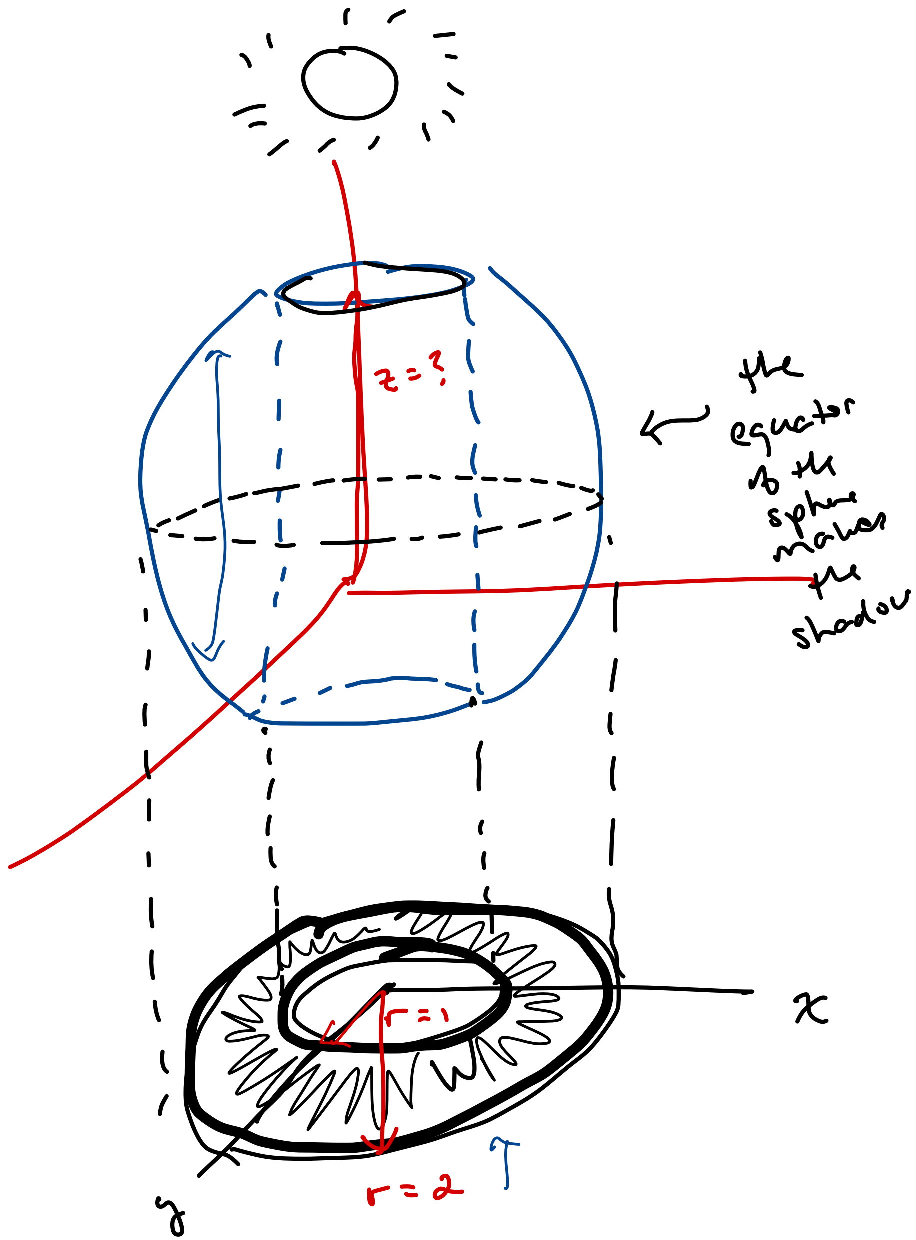
z - simple

On the sides, the z-lines go from bottom to top half. In the middle, the z-lines go from bottom half to flat top. The top function changed! **NOT** z-simple



Let Ω be the region bounded by $x^2+y^2+z^2=4$ and $x^2+y^2=1$. Find the "shadow" in the xy-plane.



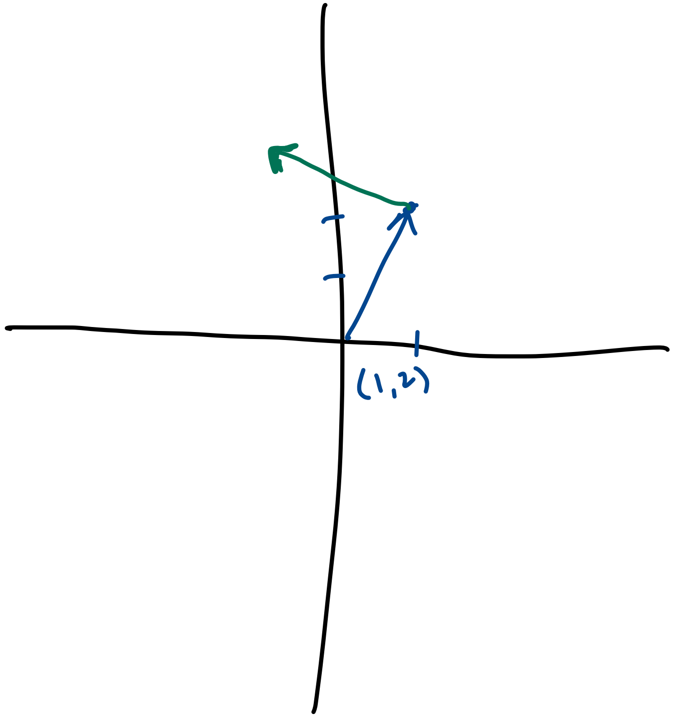


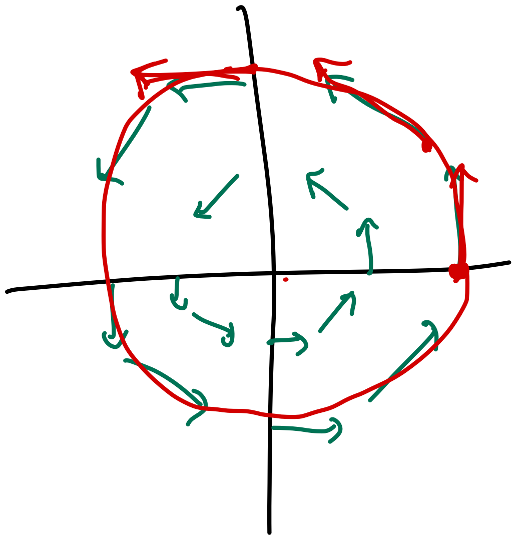
The shadow the annulus between the circles
$x^2+y^2=1$ and
$x^2+y^2=4$

2. Find the flow lines of the vector field $F(x, y) = (-y, x)$.

$F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$F(1,2) = (-2,1)$





flow line = follow the arrows

Recall a flow line has definition $c(t)$ such that

$$c'(t) = F(c(t)).$$

velocity

(green arrow) vector field value

KNOW

$$c(t) = (x(t), y(t)) \qquad F(c(t)) = F(x(t), y(t))$$

$$c'(t) = (x'(t), y'(t)) \qquad = (-y(t), x(t))$$

$$x'(t) = -y(t) \qquad y(t) = -x'(t)$$

$$y'(t) = x(t) \quad \Longrightarrow \quad x(t) = (-x'(t))' = -x''(t)$$

$$y(t) = -y''(t)$$

don't need to know!

$$\Longrightarrow \quad x(t) = a\cos(t)$$
$$y(t) = a\sin(t)$$

$\Longrightarrow$ circle!

Guess

$$x(t) = -x''(t) \longrightarrow x + \frac{d^2x}{dt^2} = 0 \qquad x = e^{rt}$$

$$\cancel{e^{rt}} + r^2 \cancel{e^{rt}} = 0 \quad \Rightarrow \quad r^2 + 1 = 0$$

$$r = \pm i$$

$$x(t) = e^{\pm it} = \underline{\cos(t)} \pm i\underline{\sin(t)}$$

3. Let $F(x,y,z) = (xz, e^y, x+y+z)$. (a) Which of the following are well-defined, $\nabla \cdot (\nabla \times F)$ or $\nabla \times \nabla F$. (b) Find $\nabla \times F$ and $\nabla \cdot F$.

Div	Grad	Curl
$\nabla \cdot F$	∇f	$\nabla \times F$
$\nabla \cdot$: vector field $\longrightarrow$ scalar function	∇ : scalar function $\longrightarrow$ vector field	$\nabla \times$: vector field $\longrightarrow$ vector field
$\nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$	$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$	$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

4. Suppose a wire can be parametrized as the intersection of the plane $z = y+2$ and $x^2+y^2 = 4$. Suppose the mass density function is given by $m(x, y, z) = z(x^2 + y^2 + 1)$. Find the total mass of the wire.