MATH 4242 Final Exam Study Guide

New Topics

- Graph (2.6)
- Digraph, directed graph (2.6)
- Connected Graph (2.6)
- Path (2.6)
- Circuit (2.6)
- Independent circuit (2.6)
- Incidence Matrix (2.6)
- Euler characteristic (2.6, lecture notes)
- Minimization of quadratics (5.2)
- Closest point in a subspace (5.3)
- Least squares solution (5.4)
- Least squares linear and polynomial data fitting (5.5)
- Hom Vector space (7.1)
- Dual Space (7.1)
- Linear Operators (7.1)
- Principle of Superposition (7.4)
- Solving differntial equation with linear operators (7.4)
- Eigenvalue, eigenvector (8.2)
- Characteristic polynomial (8.2)
- Eigenspace (8.2)
- Algebraic multiplicity (8.2, lecture notes)
- Eigenvector basis (8.3)
- Complete Eigenvalue (8.3)
- Diagonalizable, Diagonalization (8.3)
- Invariant subspaces (8.4)
- Spectral decomposition (8.5)
- Incomplete matrix (8.6)
- Unitary matrix (8.6)
- Schur decomposition (8.6)
- Jordan chain (8.6)
- Generalized eigenvector (8.6)

- Jordan block matrix (8.6)
- Jordan decomposition (8.6)
- Singular values (8.7)
- Singular value decomposition (8.7)
- Pseudoinverse (8.7)
- Linear iterative system (9.1)
- Globally asymptotically stable (9.2)
- Convergent matrix (9.2)
- Spectral Radius (9.2)
- Fixed points (9.2)
- Stable fixed point (9.2)
- Finding fixed points (9.2)
- Calculating limited behavior of iterative system (9.2)
- L^∞ norm and spectral radius (9.2)
- Markov Process (9.3)
- Translation matrix (9.3)
- Regular translation matrix (9.3)
- Probability Vector (9.3)
- Unique probability eigenvector of regular translation matrix (9.3)

Theorems

- Proposition 2.51 (2.6)
- Theorem 2.53 (2.6)
- Theorem 2.54 (2.6)
- Theorem 5.2 (5.2)
- Theorem 5.5 (5.3)
- Theorem 5.7 (5.3)
- Theorem 5.11 (5.4)
- Theorem 7.5 (7.1)
- Theorem 7.10 (7.1)
- Theorem 7.30 (7.4)
- Theorem 7.38 (7.4)
- Theorem 7.43 (7.4)
- Theorem 8.3 (8.2)
- Proposition 8.10 (8.2)

- Proposition 8.12 (8.2)
- Proposition 8.13 (8.2)
- Theorem 8.21 (8.3)
- Theorem 8.30 (8.4)
- Theorem 8.32 (8.5)
- Theorem 8.35 (8.5)
- Theorem 8.38 (8.5)
- Theorem 8.45 (8.6)
- Theorem 8.57 (8.6)
- Theorem 8.63 (8.6)
- Lemma 8.68 (8.6)
- Theorem 9.4 (9.1)
- Theorem 9.11 (9.2)
- Theorem 9.12, Theorem 9.14 (9.2)
- Proposition 9.17 (9.2)
- Proposition 9.22 (9.2)
- Theorem 9.30 (9.3)

Exam 1 Topics

- $m \times n$ matrix
- ij^{th} entry in a matrix A, a_{ij}
- row vector vs. column vector
- zero matrix
- identity matrix
- diagonal matrix
- elementary matrix
- upper/lower triangular matrix
- regular matrix
- LU decomposition
- nonsingular matrix
- permutation matrix
- permuted LU decomposition
- matrix inverse
- matrix transpose
- Theorem 1.18

- Lemma 1.19 - Lemma 1.21
- Theorem 1.28
- LDV decomposition
- Lemma 1.32
- symmetric matrix
- Theorem 1.34
- row-echelon form
- pivots
- matrix rank
- basic variables, free variables
- Theorem 1.45
- determinant of a matrix
- Theorem 1.50
- Lemma 1.51
- Theorem 1.52
- vector spaces
- $\mathbb{R}^n$, polynomials, $C^0(\mathbb{R})$
- subspaces
- linear combination
- span
- linearly independent
- linearly dependent
- Theoerem 2.21
- rank of a matrix
- basis
- Theorem 2.29
- Theorem 2.31
- Lemma 2.34
- kernel
- image
- superposition principle
- Proposition 2.41

Exam 2 Topics

- inner product on a real vector space (3.1)
- dot product on $\mathbb{R}^n$ (3.1)
- weighted dot product $\mathbb{R}^n$ (3.1)
- inner product on function vector spaces (3.1)
- norm from an inner product (3.1)
- Cauchy-Schwartz inequality (3.2)
- Triangle inequality (3.2)
- orthogonal vectors (3.2, 4.1)
- angle between two vectors (3.2)
- norm in general (3.1, 3.3)
- L^1, L^2, L^∞ norms on $\mathbb{R}^n$ and $C^0[a, b]$ (3.3)
- unit vectors (3.3)
- unit spheres (3.3)
- equivalence of norms (3.3)
  - just the statement of Thm 3.17, nothing beyond that
- matrix L^∞ norm (3.3)
- positive definite matrix, positive semi-definite matrix (3.4)
- quadratic form $x^T K x$ (3.4)
- Gram matrix (3.4)
- complex number (3.6)
- complex conjugate (3.6)
- complex inner product space (3.6)
- orthogonal and orthonormal bases (4.1)
- Gram-Schmidt (4.2)
- alternate Gram-Schmidt (4.2)
- orthogonal matrix (4.3)
- QR factorization (4.3)
- vector orthogonal to a subspace (4.4)
- orthogonal projection (4.4)
- orthogonal subspaces (4.4)
- orthogonal complement $W^\perp$ (4.4)
- cokernel, coimage of a matrix (2.5)
- linear function $T : V \to W$ (7.1)
- change of basis formula (7.2)

- Cauchy-Schwarz Inequality, Thm 3.5
- Triangle Inequality, Thm 3.9
- Theorem 3.17
- Theorem 3.20
- Def 3.23, Theorem 3.24
- Theorem 3.27
- Proposition 3.31
- Theorem 3.34
- Lemma 4.2
- Proposition 4.4, Theorem 4.5
- Theorem 4.7, Theorem 4.9
- Proposition 4.19, Lemma 4.22, Proposition 4.23
- Theorem 4.32
- Proposition 4.40, Proposition 4.41, Proposition 4.44
- Theorem 4.45
- Theorem 4.49
- Definition 7.1
- Theorem 7.5
- Example 7.19 (change of basis formula)
  - See lecture notes from 7-7 to see the theorem version of the change of basis formula