Textbook: 3.4.1abe, 3.4.5a, 3.4.10, 3.4.22ab (just i and ii), 3.4.25, 3.4.30, 3.6.4, 3.6.14, 3.6.26abc, 3.6.40a

Solution (3.4.30). Part (b) implies part (a), since we know that positive definite matrices K are both symmetric and nonsingular.

Let S be symmetric and nonsingular. Then we show that S^2 is positive definite. We can simplify $x^T S^2 x$ as follows.

$$x^T S^2 x = x^T SSx = x^T S^T Sx = (Sx)^T Sx = ||Sx||^2$$

Since S is nonsingular, then $Sx = 0$ only when $x = 0$. Therefore $||Sx||^2 > 0$ whenever $x \neq 0$, and we have shown that $x^T S^2 x > 0$ when $x \neq 0$. Thus S^2 is positive definite.

This is like how real numbers squared are always positive.

Solution (3.6.26b). There's a couple of ways to do this, but since all of chapters 1 and 2 are the same for complex numbers, we can either row reduce a matrix or compute the determinant. Put these vectors into the columns of a matrix

$$A = \begin{pmatrix} 1+i & 2 \\ 1 & 1-i \end{pmatrix}.$$

Then the first step of row reduction would be $r_2' = r_1 - (1+i)r_2$. But while multiplying $1+i$ times row 2, you notice that $1(1+i) = 1+i$ and $(1+i)(1-i) = 2$. So you get the matrix

$$\begin{pmatrix} 1+i & 2 \\ 0 & 0 \end{pmatrix}$$

after just one step. This makes the rows of the matrix dependent, and therefore the columns are dependent too! In particular

$$(1-i)\begin{pmatrix} 1+i \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1-i \end{pmatrix}.$$

Another solution would be to just take the determinant of A. Indeed

$$\det A = (1+i)(1-i) - 2 = 2 - 2 = 0.$$

Thus the vectors are dependent.