Textbook: 4.2.1a, 4.3.21, 4.3.27ac, 4.4.3, 4.4.10abc, 4.4.12a, 4.4.21, 4.4.29bcd, 7.1.1, 7.1.3, 7.1.19a-h, 7.1.25

Solution (4.4.21). First, note that $0 \subset V^\perp$, since $\langle v, 0 \rangle = 0$ for all $v \in V$. Conversely, assume $\langle w, v \rangle = 0$ for all $v \in V$. Then in particular $\langle w, w \rangle = \|w\|^2 = 0$. But by positivity of inner products, $\|w\|^2 = 0$ implies that $w = 0$. Therefore $V^\perp = \{0\}$.

Solution (4.3.27a). To find the QR decomposition, we do alternate Gram-Schmidt on the columns of A. So let $w_1 = (1, 2)$ and $w_2 = (-3, 1)$.

$$\begin{aligned}
r_{11} &= \|w_1\| = \sqrt{5} \\
u_1 &= w_1 / \|w_1\| = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\
r_{12} &= \langle w_2, u_1 \rangle = \frac{-1}{\sqrt{5}} \\
r_{22} &= \sqrt{\|w_2\|^2 - r_{12}^2} = \frac{7}{\sqrt{5}} \\
u_2 &= \frac{1}{r_2 2}(w_2 - r_{12} u_1) = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix}
\end{aligned}$$

Therefore the QR decomposition is $\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \frac{-1}{\sqrt{5}} \\ 0 & \frac{7}{\sqrt{5}} \end{pmatrix}$.