

All inner products on IR" have the form  $\langle \vec{\chi}, \vec{\eta} \rangle = \times^{T} K \eta \quad \text{where}$   $\langle \vec{\chi}, \vec{\eta} \rangle = \langle e_{1}, e_{2} \rangle \quad ... \quad \langle e_{1}, e_{n} \rangle$   $\langle e_{1}, e_{n} \rangle \quad ... \quad \langle e_{n}, e_{n} \rangle$ 

in particular if kij = ナル イズ·ダン = こ と; ス·ソ; all inner padents on Ri Josh like shis!

If all line poducts have the form 
$$(x,y) = x^T ky$$
,

Which symmetric matrices  $K$  yield an iner product?

If  $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  the corresponding three poduct is

the determinant

If 
$$K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
 the corresponding inner product is

the dot groduct!

 $\vec{\chi} = (\chi_1, \chi_2)$ 
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 $\vec{\chi} = (\chi_2, \chi_2)$ 

$$= (x, x_{2}) \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} = x_{1}y_{1} + x_{2}y_{2}$$

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 $= (x, x_2)(y_1) = x_1y_1 + x_2y_2$ 

The I = [];) Yields an isom product!

We say K is positive definite H is Symmetric XTKx >0 for all x ≠0. q(x) = xT Kx is called the quadratic form associated to K. If K is positive definite, then  $\langle \vec{\chi}, \vec{y} \rangle = \langle \chi^T K y \rangle$  is an inner product.

If K is not positive duf, ut's not an inner product.

Bilinearity. Symmetry. positivity

Positivity

when K is pos duf.

$$= (cx + dy)^{T} K^{2}$$

$$= (cx^{T} + dy^{T}) K^{2}$$

$$= (cx^{T} k + dy^{T} k)^{2}$$

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$$= cx^{T} K^{2} + dy^{T} K^{2} = (cx^{2}, 2) + dcy^{2}$$

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2) Symmetry,  $K = (cx^{2}, 2) + dcy^{2}$ 

 $\langle x,y\rangle = x^TKy = x^TK^Ty = (Kx)^Ty$ 

りく(な+ぬ,毛)

 $(y^T Kx)^T = (Kx)^T y^{TT} = (Kx)^T y$ (3,x) =  $\mathbb{R} \qquad |x| \qquad (x^7 kx)^T = x^7 kx$ So (x,y) = (y,x) 3) Positivity if K is a positive afficial matrix, by all XTKX70; to all X70. sity writin!  $\langle \vec{x}, \vec{x} \rangle = \frac{1}{x^{T} K x^{T} x^{T}} \int_{-\infty}^{\infty} |x|^{2} dx$ Same! Positivity axion! So K hung pos ded is exactly saying that (x,y) = XT Ky satisfied axim

Are 
$$\begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix}$$
 and  $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$  position authority?

Are  $\langle x,y \rangle = \chi T \begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix} \mathcal{J}$  and  $\langle x,y \rangle = \chi T \begin{pmatrix} 12 \\ 12 \end{pmatrix} \mathcal{J}$ 

Are  $\langle x,y \rangle = \chi T \begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix} \mathcal{J}$  and  $\langle x,y \rangle = \chi T \begin{pmatrix} 12 \\ 12 \end{pmatrix} \mathcal{J}$ 

Are  $\langle x,y \rangle = \chi T \langle x \rangle = \langle x, x_2 \rangle \begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} x_2 \\ x_2 \end{pmatrix}$ 

Of  $\langle x \rangle = \chi T \langle x \rangle = \langle x, x_2 \rangle \begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ 

The expression of the expression

 $= (x_1 + z_2) \begin{pmatrix} 4x_1 - 2x_2 \\ -2x_1 + 3x_2 \end{pmatrix}$ 

$$= \frac{4x_1^2 - 4x_1x_2 + 3x_2^2}{-4x_1x_2 + 3x_2^2} + 2x_2^2$$

$$= (2x_1 - x_2)^2 + 2x_2^2$$
 Complising the square strategy.

$$2x, -x = 0$$

$$2x_1 = 0 \quad \neq 1 \quad (x_1 x_1) \neq 0$$

$$2x_{1} = 0 \quad \neq 3 \quad (x_{1} x_{1})$$

$$1 \quad 4^{-2} \quad \Rightarrow \text{ since duting}$$

$$2x_{1} = 0 \neq (x_{1}x_{1})$$

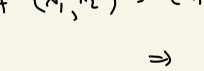
$$\begin{cases} 1 & 4 & -2 \\ -2 & 3 \end{cases}$$
 is positive dufinite!

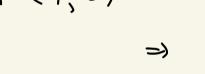
$$2x_1 = 0 \qquad \neq 3 \qquad (x_1 x_1) \neq 0$$

So (-23) is positive definite!

 $\langle \chi, \chi \rangle = \chi^{T} \begin{pmatrix} -2 & 3 \end{pmatrix} \chi \quad \text{is an inom product}$ 

= 4x17, -2x24, -2x,42 + 3x2





(12) is not positive definite and

 $(2x,y) = xT(2^{2})y = x_1y_1 + 2x_1y_2 + 2x_2y_3 + x_2y_2$ 

is not an inner product.

A 2 = 2 symmetric matrix is position definite if our only if ac-52 10 "astronate " where  $K = (a)^b$ . aux K

$$\begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix} = \begin{pmatrix} -2 & 7 & 7 & 7 \\ -2 & 3 & 7 \end{pmatrix}$$

$$\implies positive def.$$

(12) -> 1.1-2.2 = -3 <0 not positive def. Proof Friday ...

let I be any vector space up an inn product Gran matrices let ~,... ~, EV. the Gram matrix of V, -- Vu In he  $K = \begin{pmatrix} \langle v_1 v_1 \rangle & \dots & \langle v_k v_k \rangle \\ \langle v_1 v_k \rangle & \dots & \langle v_k v_k \rangle \end{pmatrix} \quad \text{matrix}$ Thm -v, -- vu one independent iff K is position definite.

Pary also a Friday!

Ex Show that cos(x), cos(2x), cos(3x) are independent functions

or (°[0,217]. There is no try identity between these L' since product or Co.

Define  $\langle f,g \rangle = \int_{2\pi}^{2\pi} f(x) g(x) dx$ < ((3x))</pre> (1x1) W(1x1)

\( \ous 3x, \ous 3x \)
\( \ou 3x \)
\( \ous 3x \)
\( \ou 3x \) \(
\left\) \(
\lef ( LOS 24, W 24) ( wszr, wzx)

$$\langle x_{1}x^{2}, x_{2}x^{2} \rangle = \int_{0}^{2\pi} x_{1}(x) \cos(x) \sin(x) dx = 0$$

$$\langle x_{1}x^{2}, x_{2}x^{2} \rangle = \int_{0}^{2\pi} x_{1}(x) \cos(x) dx = 0$$

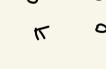
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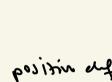
$$\langle x_{1}x^{2}, x_{2}x^{2} \rangle = \int_{0}^{2\pi} x_{2}x^{2} \cos(x) dx = 0$$

= 20 m/5x) cox(3x) gx = 0 رمرورم , دمروم

$$K = \begin{pmatrix} 0 & \pi & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{pmatrix}$$
 u position ouf.







K is got def since 
$$g(x) = \chi T K \chi$$

$$(\chi_1 \chi_2 \chi_3) \begin{pmatrix} \sigma & \sigma & \chi_1 \\ \sigma & \sigma & \chi_2 \end{pmatrix} = \pi \chi_1^2 + \pi \chi_2^2 + \pi \chi_3^2 > 0.$$

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the K is possely lift all det (Ki) >0.

Pt LU auropisition

$$H(f) = \begin{bmatrix} \frac{2^{2}}{3x^{2}} & \frac{2^{2}}{3x^{2}} \\ \frac{2^{2}}{3x^{2}} & \frac{2^{2}}{3y^{2}} \end{bmatrix}$$

$$(x_{0}, y_{0})$$

H(f) is pos M.

Is this in in product?

 $\langle \lambda' M \rangle^T = \frac{A}{\Gamma} \left( \| \lambda + M \|^2 - \| \Lambda - M \|^2 \right)$ ulere 11-11/2 is the L' norm.

| vll\_ = [ |vi| = |vi| + ...+ |vn| Is then an inner product <-,->1 such that  $||v||_1 = \int \langle v, v \rangle$ ?

(a) If L-,->2 were to exist. It have to have

