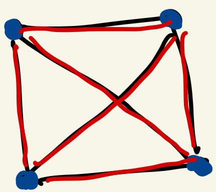
HW 9 due this Friday! (has before the break material)

Section 2.6 Graph Theory

Define: A graph G is a collection of vertices and edges. A vertex v is represented by a dot and an edge e is a line segment between to vertices.

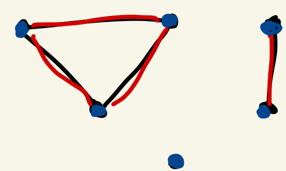
Ex



4 vertices

6 edges

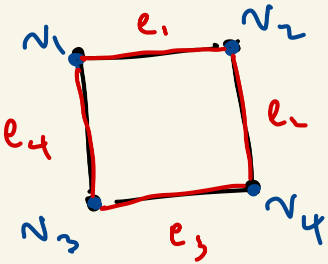
Ex

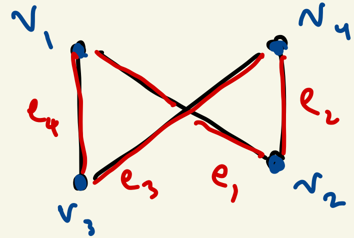


6 vertices

4 edges

Here are 2 different ways to draw the same graph.





All that matters is which edges are connected to which vertices!

Edges have have no direction!

$G = (E, V)$ $\quad V = \{v_1 \ldots v_n\}$ $\quad E =$ set of pairs from V

$V = \{v_1, v_2, v_3, v_4\}$ $\quad E = \{ \{v_1 v_2\}, \{v_2 v_4\}, \{v_3 v_4\}, \{v_1 v_3\} \}$

Typically: Only a finite amount of edges and vertices

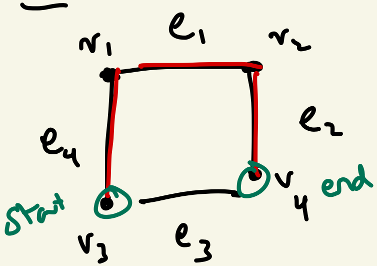
If you want to consider infinite graphs, you might assume only finitely many edges connected to a vertex.

∞ amount

We will only consider finite graphs!

Def: A path within a graph G is a series of edges $e_1, \dots e_n$ such that e_i and e_{i+1} share a vertex.

Ex

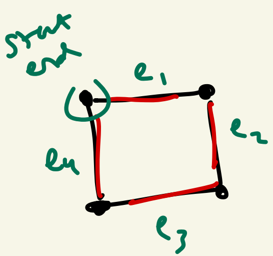


e_4, e_1, e_2 is a path

e_4 and e_1 share vertex 1 in common

etc

Not a circuit



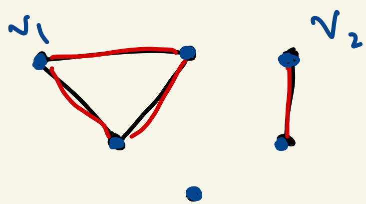
$e_1 e_2 e_3 e_4$ is another path

This is a circuit.

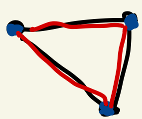
Def: A path in G is called a circuit if it starts and ends at the same vertex.

Def: We say a graph is connected if there exist a path from any vertex to any other vertex.

Ex

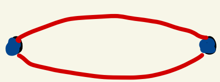


This is NOT a connected graph. There's no path from v_1 to v_2.



This smaller graph is connected!

Note:



In some books, this would be considered a graph. In others this would be a multigraph.

We won't consider more than 1 edge between two vertices.

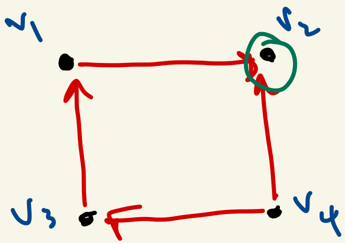
<u>Digraphs</u> Now we add <u>directions</u> to our edges.

<u>Def</u>: A digraph is a graph but with additional information about where edges start and end.

$e = (v_i, v_j)$ Digraph

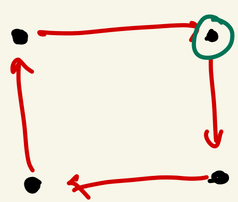
$e = \{v_i, v_j\}$ graph

In digraphs, edges are represented by arrows.



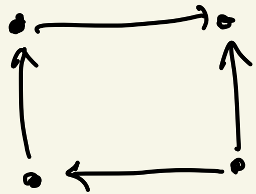
v_2 has arrowheads

Both of these are connected.



v_2 only has 1

These are two different digraphs.



is <u>connected</u> since the underlying graph is connected.

Despite the fact there's no path out of v_2.

<u>Def</u> Given a <u>digraph</u> G, we can associate 2 vector spaces to G. Suppose G has vertices $v_1 \ldots v_n$, edges $e_1 \ldots e_m$

C_1 = all $\boxed{\text{formal}}$ linear combinations of $e_1 \ldots e_m$.

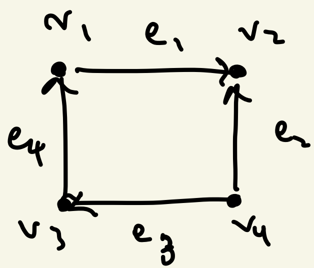
$$e_1 = 1e_1 + 0e_2 + \ldots + 0e_m \in C^1$$

$$2e_1 + \tfrac{1}{2}e_2 + 5e_3 \in C^1.$$

formal means that $e_1 \ldots e_m$ is a "basis" in an abstract sense

If somehow $e_1 .. e_m$ were a basis of a vector space, what would that vector space look like? You get C_1.

C_0 = all formal linear combinations of $v_1 .. v_n$.

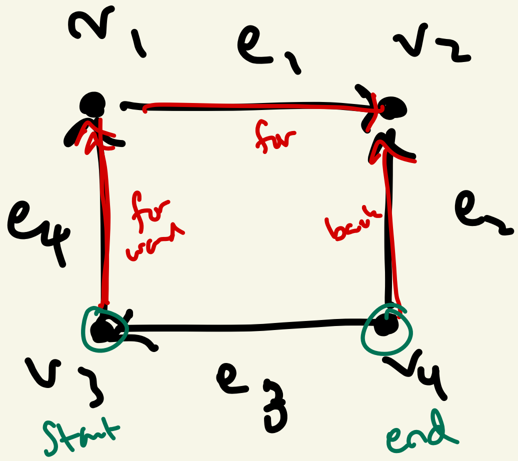


$C_1 = \mathrm{Span}(e_1, e_2, e_3, e_4)$

$C_0 = \mathrm{Span}(v_1, v_2, v_3, v_4)$

We can just turn vertices and edges in basis vectors of a vector space arbitrarily.

We can write path / circuits as certain linear combinations of edges.



e_4, e_1, e_2 is a path.

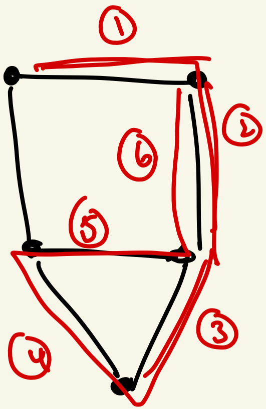
$$e_4 + e_1 - e_2$$

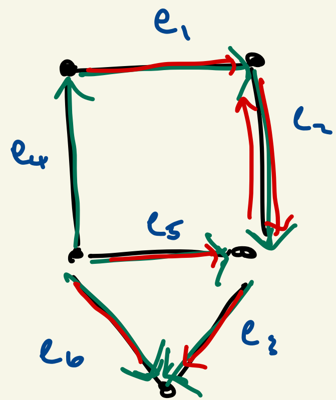
backwards!

$+$ represents going forward

$-$ represents going backward.

This path $e_4 + e_1 - e_2 \in C_1$ lives in C_1.





So the path as a linear combination is

$$e_1 + \cancel{e_2} + e_3 - e_6 + e_5 - \cancel{e_2}$$

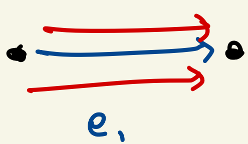
$$= e_1 + e_3 + e_5 - e_6$$

$$= e_1 + e_5 + e_3 - e_6$$

$$= -e_6 + e_1 + e_3 + e_5$$

1) Label edges

2) add some directions | make this a digraph



$2e_1$

$$-x^{*T}f < 0$$

$$x^* = K^{-1}f$$

$$-x^{*T}f = -f^T\underbrace{(K^{-1})^T}_{\text{pos def}}f < 0$$

$$-f^T(K)^{-1^T}f < 0 \quad \text{for } K \text{ pos def}$$