Applied Linear Algebra

Derivatives

product

chain rule

by parts

u-sub

Integrals

diff

multi

linear alg

Linear Alg

row reduction

computational tool

eigenvectors

linear ind.

pos def

inner products

Vectors, Matrices, Row reduction

Linear Systems of Equations

$$a_{11}x_1 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$$

n variables
m equations

$a_{11}\ a_{21} \dots\ a_{mn}$ are all the coefficients.

$x_1 \dots x_n$ are the variables

$b_1 \dots b_m$ are the coefficients by themselves.

2 variables 2 equations

$$3x + 2y = 1$$
$$1x - 1y = 2$$

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$3x + 2y = 1$$
$$3\ (x - y = 2)$$

$$3x + 2y = 1$$
$$-3(x - y = 6)$$

$$\cancel{3x} + 2y = 1$$
$$-\quad \cancel{3x} - 3y = 6$$

$$3x + 2y = 1$$
$$+\quad -3x + 3y = -6$$

$$5y = -5$$
$$y = -1$$
$$x - (-1) = 2$$
$$x = 1$$

$$\boxed{3}x + \boxed{2}y = 1$$
$$\boxed{1}x - \boxed{1}y = 2$$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \vec{x} = \vec{b}$$

Linear System in matrix form

m equations n variables

A $m \times n$ matrix m rows n columns

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

$m \times \cancel{n}$ $\quad$ $\cancel{n} \times 1$ $\quad$ $m \times 1$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \begin{aligned} 3x + 2y &= 1 \\ x - y &= 2 \end{aligned}$$

$$A \vec{x} = \vec{b}$$

Definition of $A\vec{x}$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \cdot x + 2 \cdot y \\ 1 \cdot x + (-1) \cdot y \end{bmatrix}$$

$$= \begin{bmatrix} 3x + 2y \\ 1x - 1y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

<u>In general</u>, If A is an $m \times n$ matrix (m rows, n columns) then you can multiply an n-vector.

$$\vec{v} = (v_1, v_2, \ldots, v_n) = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

<u>In general</u>, If A is an $m \times n$ matrix (m rows, n columns) then you can multiply an n-vector.

$$\vec{v} = (v_1, v_2, \dots, v_n) = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$\begin{bmatrix} \boxed{a_{11} \ \cdots \ a_{1n}} \\ \vdots \\ \boxed{a_{m1} \ \cdots \ a_{mn}} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} a_{11} \cdot v_1 + a_{12} \cdot v_2 + \dots + a_{1n} \cdot v_n \\ \vdots \\ a_{m1} \cdot v_1 + \dots + a_{mn} \cdot v_n \end{bmatrix}$$

This the definition!

Ex

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\,(-1) + 2\cdot 3 \\ 1\cdot(-1) + (-1)\cdot 3 \\ 1\cdot(-1) + 0\cdot 3 \end{bmatrix}$$

$\underset{\text{rows}}{3} \times \underset{\text{columns}}{\cancel{\boxed{2}}}$ $\qquad \cancel{2} \times \underset{\text{columns}}{1}$ $\qquad 3 \times 1$

has to be 2 in order to multiply

$$= \begin{bmatrix} 5 \\ -4 \\ -1 \end{bmatrix}$$

Next time

Matrix Multiplication

Row reduction

Matrix Inverses ...

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$3 \times \underline{3}$ $\underline{3} \times 1$

§ 1.4 LU decomposition

§ 1.5 Inverses