Permuted LU decomposition.

If A is regular

then $A = LU$

$$\begin{pmatrix} a_{11} & & \\ \downarrow & a_{22} & \\ & \downarrow & \ddots \end{pmatrix} = \begin{pmatrix} 1 & & & \\ * & 1 & & \\ * & * & 1 & \\ * & * & * & 1 \end{pmatrix} U$$

<u>Definition</u> Define S_n to be the set of all permutations of n elements

$n = 3$

$$S_3 = \left\{ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}, \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}, \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right\}$$

$$P\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} b \\ c \\ a \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

If we include the swap (i,j) row operation

$$\underline{P}A = \underbrace{LU}_{\text{same as before}}$$

keeps track of all the row swaps

same as before

L keeps track of $cr_i + r_j$

$A \longrightarrow U$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \xrightarrow{\text{swap } 1,2} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

$$\xrightarrow{r_2' = 5r_1 + r_2} \begin{pmatrix} 3 & 4 \\ 16 & 22 \end{pmatrix}$$

switch order!

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \xrightarrow{r_2' = 5r_1 + r_2} \begin{pmatrix} 1 & 2 \\ 8 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{swap } 1,2} \begin{pmatrix} 8 & 14 \\ 1 & 2 \end{pmatrix}$$

$cr_i + r_j \to \text{swap} \to c' r_i + r_j$

$\to \text{swap}$

$\neq$ swaps $\longrightarrow cr_i + r_j$

So interchanging the row operations changes the resulting row reduced matrix!

when computing $PA = LU$, L is not quite as easy to compute as before,

You have keep track of permuting L as well.

Recall that every row operation is an elementary matrix.

$E \rightsquigarrow cr_i + r_j \qquad P \rightsquigarrow \text{swap}$

$$U = E_7 E_6 P_5 \overbrace{E_4 E_3 E_2} P_1 A$$

??

$$= \left(E_7 E_6 \boxed{\tilde{E}_4 \tilde{E}_3 \tilde{E}_2}\right)\left(P_5 P_1 A\right)$$

Lower triangular

The $\tilde{E}_4 \; \tilde{E}_3 \; \tilde{E}_2$ means that you have to find some new row operations to make things work.

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & 6 & 2 & -1 \\ 1 & 1 & -7 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$$

<u>Ex</u> Compute permuted LU decomposition of A

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ \boxed{3} & 6 & 2 & -1 \\ 1 & 1 & -7 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\xrightarrow{-3r_1+r_2} \quad A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 5 & -1 \\ \boxed{1} & 1 & -7 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & & \\ 3 & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$\cancel{-3r_1+r_3}$

$$\xrightarrow{-r_1+r_3} \qquad \longrightarrow \quad -r_1+r_2$$

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & -1 & -6 & 2 \\ \boxed{1} & -1 & 2 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & & \\ 3 & 1 & & \\ 1 & & 1 & \\ & & & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\xrightarrow{-r_1+r_4}$$

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & \boxed{0} & 5 & -1 \\ 0 & \boxed{-1} & -6 & 2 \\ 0 & \boxed{-3} & 3 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & & \\ 3 & 1 & & \\ 1 & & 1 & \\ 1 & & & 1 \end{pmatrix}$$

If A was regular green box would be nonzero.

$$P = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\xrightarrow{\text{swap }(2,3)}$$

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & \boxed{-1} & -6 & 2 \\ 0 & 0 & 5 & -1 \\ 0 & \boxed{-3} & 3 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ 3 & & 1 & \\ 1 & & & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{pmatrix}$$

$$\xrightarrow{-3r_2 + r_4}$$

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & -6 & 2 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & \boxed{21} & -5 \end{pmatrix} \quad L = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ 3 & & 1 & \\ 1 & 3 & & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix}$$

$$\xrightarrow{\boxed{-\frac{21}{5}} r_3 + r_4}$$

$$U = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & -6 & 2 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & -4/5 \end{pmatrix} \quad L = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ 3 & & 1 & \\ 1 & 3 & \frac{21}{5} & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix}$$

The permuted LU decomp. is

$$PA = LU$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & 6 & 2 & -1 \\ 1 & 1 & -7 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 3 & \frac{21}{5} & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & -6 & 2 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & -\frac{4}{5} \end{pmatrix}$$

Ex

$$\begin{pmatrix} \boxed{0} & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\xrightarrow{\text{swap } 1,3} \boxed{\begin{pmatrix} 1 & 1 & 2 \\ \boxed{3} & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}} \qquad L = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \qquad P = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$$

$$\xrightarrow{-3r_1 + r_2} \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & -5 \\ 0 & \boxed{1} & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & \\ 3 & 1 & \\ & & 1 \end{pmatrix} \qquad P = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$$

$$\xrightarrow{\frac{1}{3}r_2 + r_3} \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & -5 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} \qquad L = \begin{pmatrix} 1 & & \\ 3 & 1 & \\ 0 & -\frac{1}{3} & 1 \end{pmatrix} \qquad P = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$$

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} \boxed{0} & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ 3 & 1 & \\ & -\frac{1}{3} & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & -5 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

> If A is regular
>
> $A = LU$ is unique

If A is nonsingular

$PA = LU$ is <u>not</u> unique.

<u>Def</u> A $n \times n$ matrix is <u>nonsingular</u>

if it has <u>n</u> pivots, i.e.

you can reduce to a U w/ nonzero diagonal entries, using any row opr.

<u>Def</u> A <u>matrix</u> is nonsingular if it's <u>invertible</u>.

LDV decomposition:

Let A be a regular matrix.

$$A = LU$$

<u>Ex</u> let $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & & \\ 3 & 1 & \\ & -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & \boxed{-3} & -5 \\ 0 & 0 & \boxed{\frac{1}{3}} \end{pmatrix}$$

In order to make $-3 \longrightarrow 1$

$\frac{1}{3} \longrightarrow 1$

$$\boxed{r_2' = -\frac{1}{3} r_2 \qquad r_3' = 3 r_3}$$

$$A = \begin{pmatrix} 1 & & \\ 3 & 1 & \\ & -\frac{1}{3} & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix}}_{U}$$

$$= LDV$$

<u>Def</u> An upper Δ matrix is uniuppertriangular if $u_{ii} = 1 \ \forall i$.

A lower Δ matrix is unilower Δ if $l_{ii} = 1 \ \forall i$.

The LDV decomp writes A as a product of unilower Δ · diagonal · uniupper Δ.

① $\xrightarrow{\text{swap } 1,3}$ $\xrightarrow{r_1 = 2r_1 + r_2}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 2 & -1 & 1 \\ -1 & 4 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & 4 & 2 \\ 2 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} -1 & 4 & 2 \\ 0 & 7 & -3 \\ 0 & 0 & 1 \end{pmatrix} = U$$

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 2 & -1 & 1 \\ -1 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & & \\ 2 & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} -1 & 4 & 2 \\ 0 & 7 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

② Let P, Q be permutation matrices, QP is also a permutation.

$$QP\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

$$P \rightsquigarrow \sigma \quad \begin{matrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{matrix}$$

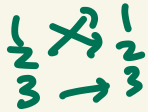
$$Q \rightsquigarrow \tau \quad \begin{matrix} 1 & 2 & \cdots & n \\ \tau(1) & \tau(2) & \cdots & \tau(n) \end{matrix}$$

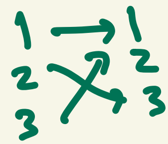
$$QP\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = Q\begin{pmatrix} a_{\sigma(1)} \\ \vdots \\ a_{\sigma(n)} \end{pmatrix}$$

$$= \begin{pmatrix} a_{\tau(\sigma(1))} \\ \vdots \\ a_{\tau(\sigma(n))} \end{pmatrix}$$

Doing 2 rearrangements in a row is another rearrangement!

Therefore QP is another permutation matrix.





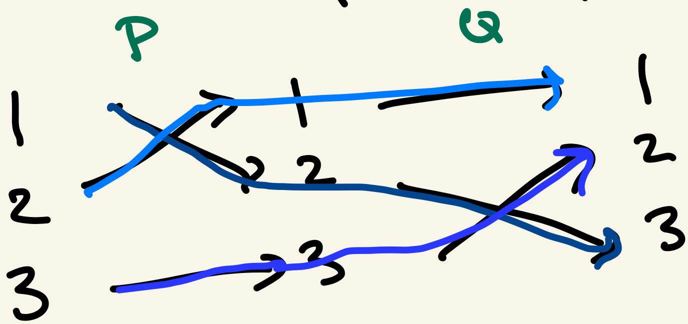
__Ex__

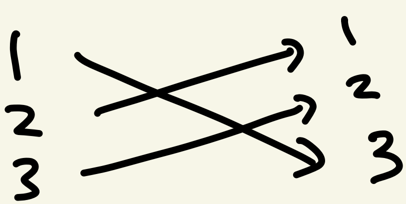
$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$QP = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

This is another P. matrix!





$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

QP

A permutation is a bijection

$$\{1, \dots, n\} \longrightarrow \{1, \dots, n\}$$

If you compose $\tau \circ \sigma$, you'll get another bijection.

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§1.5 / 1.6 Inverses, Transposes, Symmetric Matrices

<u>Define</u> An $n \times n$ matrix A is <u>invertible</u> if there exists a matrix B such that

$$AB = I_n = BA$$

$I_n \rightsquigarrow 1$

$A^{-1} \rightsquigarrow$ "$\frac{1}{A}$"

Normally we call $B = A^{-1}$.

A^{-1} is called the <u>inverse</u> of A.

Not every matrix has an inverse!

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ has no inverse!

(You can't divide by 0)

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ also has no inverse.

<u>Most</u> are invertible.

How to compute A^{-1} using row reduction.

Let A be nonsingular

$\rightarrow$ you can row reduce

$A \rightarrow I$ using swap $r_i + r_j$.

nonzero entries

We can use these to compute A^{-1}.

The inverse A^{-1} solves the matrix equation

$$AX = I. \qquad X = \begin{pmatrix} \vec{x}_1, \vec{x}_2 \dots \vec{x}_n \end{pmatrix}$$

$$\begin{pmatrix} Ax_1 & \dots & Ax_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ 0 & & \ddots & \vdots \\ \vdots & & & \\ 0 & 0 & & 1 \end{pmatrix}$$

You can solve

$$\left.\begin{aligned} A\vec{x}_1 &= \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \\ A\vec{x}_2 &= \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \\ &\vdots \\ A\vec{x}_n &= \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \end{aligned}\right\} \text{The same row reduction solves all of these at once}$$

by

row reducing augmented matrix

$$\left(A \mid I_n \right).$$

$$\Big\downarrow \text{row reduction}$$

$$\left(U \mid Y \right)$$

$$\Big\downarrow \text{back substitution}$$

$$\left(I \mid A^{-1} \right)$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$ Find the inverse of A.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\text{swap } 2,3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & \boxed{3} & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \xrightarrow{-3r_3 + r_2 = r_2'}$$

$$\left(\begin{array}{ccc|ccc} 1 & \boxed{1} & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \xrightarrow{-r_2 + r_1 = r_1'}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -1 \\ 0 & 1 & 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1 & 3 & -1 \\ 0 & -3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 3 & -1 \\ 0 & -3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

<u>Thm</u> If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

then $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

provided $ad-bc \neq 0$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Prop Matrix inverses are unique.

Pf Assume A^{-1} and B are both inverses of A. Then,

$$A^{-1} = A^{-1} I_n = A^{-1}(AB)$$

$$= (A^{-1}A)B = I_n B = B.$$

So $A^{-1} = B$. $\square$ ■

(tombstone)

(square)

Let's say we have a system of eq'ns

$A\tilde{x} = b$, and we know A^{-1}.

then $\cancel{A^{-1}A}x = A^{-1}b$

$$\tilde{x} = \underbrace{A^{-1}b}.$$

Prop If A is regular, then the LU decomp. into a unilower Δ matrix on upper Δ is unique.

Pf $A = LU$

$= \tilde{L}\tilde{U}$.

Turns out $\tilde{L}$ and $\tilde{U}$ are invertible!

$$\left(\begin{matrix} 1 & & & \\ * & 1 & & \\ * & * & 1 & \\ & & & 1 \end{matrix}\,\middle|\, I_n\right) \longrightarrow \left(I_n \,\middle|\, \tilde{L}^{-1}\right)$$

$$\left(\begin{matrix} * & * & * \\ & * & * \\ & & * \end{matrix}\,\middle|\, I_n\right) \xrightarrow[\text{substitution}]{\text{back}} \left(I_n \,\middle|\, \tilde{U}^{-1}\right)$$

$LU = \tilde{L}\tilde{U} \implies$

$\tilde{L}^{-1} L U \cancel{U^{-1}} = \cancel{\tilde{L}^{-1}\tilde{L}}\tilde{U}U$

$$\underbrace{\tilde{L}^{-1}L}_{\text{Uni lower } \Delta \text{ matrix}} = \underbrace{\tilde{U}U^{-1}}_{\text{upper } \Delta \text{ matrix}}$$

$$\begin{pmatrix} 1 & & 0 \\ & 1 & \\ * & & 1 \end{pmatrix} = \begin{pmatrix} * & & * \\ & * & \\ 0 & & * \end{pmatrix}$$

all diagonal entries are 1, and off diagonals are zero.

$$= I_n.$$

$\tilde{L}^{-1}L = I \implies$ since inverses are unique

$\tilde{L} = L$

$U\tilde{U}^{-1} = I \implies \tilde{U} = U.$ $\square$

<u>Corollary</u>: The LDU decomp. is also unique.

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§ 1.6

<u>Def</u> Let A be an $m \times n$ matrix.

Then define "A transpose", A^T to be the $n \times m$ which results from turning rows of A into columns and columns of A into rows. $(A^T)_{ij} = A_{ji}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{bmatrix}$$

<u>Def</u> An $n \times n$ matrix is <u>Symmetric</u> if $A = A^T$.

<u>Ex</u>

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$ is symmetric.

The transpose is also

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

<u>Thm</u> 1.34
Let A be a regular symmetric matrix.

Then $A = LDL^T$.

<u>Prop</u> $(AB)^{-1} = \underbrace{B^{-1}A^{-1}}$.

<u>Pf</u>

$(\cancel{B^{-1}A^{-1}})\cancel{A}B = \cancel{B^{-1}}B = I$

$AB(B^{-1}A^{-1}) = AA^{-1} = I$.

<u>Prop</u> $\underline{(AB)^T = B^T A^T}$.

<u>Pf</u>: compare elements.