# MATH 4242 Final Exam Study Guide

**New Topics**

- Graph (2.6)
- Digraph, directed graph (2.6)
- Connected Graph (2.6)
- Path (2.6)
- Circuit (2.6)
- Independent circuit (2.6)
- Boundary operator (2.6, lecture Notes)
- Minimization of quadratics (5.2)
- Closest point in a subspace (5.3)
- Least squares solution (5.4)
- Least squares linear and polynomial data fitting (5.5)
- Linear Transformation, Linear Operators (7.1) (Exercises 7.1.1, 7.1.19)
- Eigenvalue, eigenvector (8.2)
- Characteristic polynomial (8.2)
- Eigenspace (8.2)
- Algebraic multiplicity (8.2, lecture notes)
- Geometric multiplicity (8.2, lecture notes)
- Eigenvector basis (8.3)
- Complete Eigenvalue (8.3)
- Diagonalizable, Diagonalization (8.3)
- Spectral decomposition (8.5)
- Jordan chain (8.6)
- Generalized eigenvector (8.6)
- Jordan block matrix (8.6)
- Jordan decomposition (8.6)
- Matrix Exponential (10.4, lecture notes)
- Exponential Properties (10.4, lecture notes)
- Computation using Jordan form (10.4, lecture notes)

**Theorems**

- Theorem 2.53 (2.6)
- Theorem 2.54 (2.6)
- Theorem 5.2 (5.2)
- Theorem 5.5 (5.3)
- Theorem 5.7 (5.3)
- Theorem 5.11 (5.4)
- Theorem 7.5 (7.1)
- Theorem 8.3 (8.2)
- Proposition 8.13 (8.2)
- Theorem 8.32 (8.5)
- Theorem 8.35 (8.5)
- Theorem 8.38 (8.5)
- Theorem 8.57 (8.6)
- Lemma 10.2 (10.4, lecture notes)
- Formula 10.47 (10.4, lecture notes)
- Proposition 10.30 (10.4, lecture notes)

**Exam 1 Topics**

- $m \times n$ matrix
- $ij^{th}$ entry in a matrix $A$, $a_{ij}$
- row vector vs. column vector
- zero matrix
- identity matrix
- diagonal matrix
- elementary matrix
- upper/lower triangular matrix
- pivots
- $LU$ decomposition
- nonsingular matrix
- permutation matrix
- permuted $LU$ decomposition
- matrix inverse
- matrix transpose
- Theorem 1.18
- Lemma 1.19 - Lemma 1.21
- Theorem 1.28
- Lemma 1.32
- symmetric matrix
- Reduced Row Echelon Form (RREF)
- matrix rank
- Interpreting solution from RREF, free variables
- Theorem 1.45
- vector spaces
- $\mathbb{R}^n$, polynomials, $C^0(\mathbb{R})$
- subspaces, defining properties of subspaces
- linear combination
- Span$(v_1, \ldots, v_n)$
- Spanning set of a vector space
- linearly independent vectors
- linearly dependent vectors
- Basis of a vector space, basis of a subspace
- Theoerem 2.21, Theorem 2.28, Theorem 2.29

- rank of a matrix
- Fundamental Subspaces: kernel, image, cokernel, coimage
- column space, row space, null space
- Proposition 2.41
- Theorem 2.49
- Fundamental Theorem of Linear Algebra (Equivalent Conditions of Invertibility)

**Exam 2 Topics**

**Topics**

- inner product on a real vector space (3.1)
- dot product on $\mathbb{R}^n$ (3.1)
- weighted dot product $\mathbb{R}^n$ (3.1)
- inner product on function vector spaces (3.1)
- norm from an inner product (3.1)
- Cauchy-Schwartz inequality (3.2)
- Triangle inequality (3.2)
- orthogonal vectors (3.2, 4.1)
- angle between two vectors (3.2)
- norm in general (3.1, 3.3)
- $L^1$, $L^2$, $L^\infty$ norms on $\mathbb{R}^n$ and $C^0[a, b]$ (3.3)
- unit vectors (3.3)
- positive definite matrix (3.4)
- quadratic form $x^T K x$ (3.4)
- Gram matrix (3.4)
- complex number (3.6)
- complex conjugate (3.6)
- complex row reduction (3.6)
- orthogonal and orthonormal bases (4.1)
- Gram-Schmidt (4.2)
- alternate Gram-Schmidt (4.2)
- orthogonal matrix (4.3)
- QR factorization (4.3)
- vector orthogonal to a subspace (4.4)
- orthogonal projection (4.4)
- orthogonal subspaces (4.4)
- orthogonal complement $W^\perp$ (4.4)
- cokernel, coimage of a matrix (2.5)

**Theorems**

- Cauchy-Schwarz Inequality, Thm 3.5
- Triangle Inequality, Thm 3.9
- Theorem 3.27
- Proposition 3.31
- Theorem 3.34
- Lemma 4.2
- Proposition 4.4, Theorem 4.5
- Theorem 4.7, Theorem 4.9
- Proposition 4.19, Lemma 4.22, Proposition 4.23
- Theorem 4.32
- Proposition 4.40, Proposition 4.44
- Theorem 4.45