Midterm 2 Review

1. Let $R = [-1,1] \times [-1,1]$ be the unit square. Find the integral

$$\iint_R x^2 + y^2 \, dx \, dy.$$

2. Let S be the region bounded by $x = 0$, $x = 4$, $y = x^2$ and $y = 2x^2$. Find the integral

$$\iint_S x^2 - y^2 \, dx \, dy.$$

3. Let T be the region in between the parabola $x = y^2 - 1$ and $x = 2y - 1$. Find the integral

$$\iint_T xy \, dx \, dy.$$

4. Calculate the integral

$$\int_0^2 \int_0^{3x} e^{12y - y^2} \, dy \, dx.$$

5. Let $R = [-1,1] \times [-1,1] \times [-1,1]$ be the unit cube. Find the integral

$$\iiint_R x^2 + y^2 + z^2 \, dx \, dy \, dz.$$

6. Let S be the region bounded by the equations $z = x^2 + 2y^2$ and $z = 5 - x^2 - y^2$. Set up the integral

$$\iiint_S 1 \, dV = \text{volume of region } S$$

and evaluate it if you can.

7. Find the curl and the divergence of the vector field $\vec{F} = (y, z, x)$ at the point (3,2,2).

8. Find the gradient vector field of $f(x,y) = e^{x^2} - 2xy$ and verify that $\nabla \times \nabla f = 0$.

9. Evaluate

$$\int_C F \cdot ds$$

where $F(x,y) = (x^2, y^2)$ and C is the straight line from (-1,5) to (6,12).

10. Let C be the unit circle in the xy-plane of $\mathbb{R}^3$. Find

$$\int_C ydx + zdy + xdz.$$

11. Find the work done by the force $F(x,y) = (x^2 - y^2, 2xy)$ for a particle moving in counterclockwise on the square $(0,0), (a,0), (a,a)$, and $(0,a)$.

12. Let C be the curve (t, t^2, t^3) and $F(x,y,z) = (x, y^2, z^3)$. Find

$$\int_C F \cdot ds.$$

13. Consider the curve $c(t) = (-t, t^2, 2t + 3t^2)$. As a function, what is the domain and range of the curve c? What is the derivative $c'(t)$? What is the tangent line at $t = 1$?

14. Let $g(x,y,z) = x^2 + y^2 + z^2$. Use the gradient to find the tangent plane to the surface $f(x,y,z) = 1$ at $(0,1,0)$. What about at the point $(1,1,1)/\sqrt{3}$?

15. Let $c(t) = (\cos 2t, \sin 2t)$. Show that $|c(t)|$ is constant and that $c'(t)$ is perpendicular to $c(t)$ for all t. Can you think of a curve $c(t)$ where $|c(t)|$ is constant but $c'(t)$ is not perpendicular to $c(t)$?

16. Let $c(t) = (t, t^2, t - t^2)$ be the path of a particle p of mass 5 kg moving in $\mathbb{R}^3$. Calculate the total force on the particle. (I don't think that this is on the midterm but it's good practice.) Say one of the forces that acts on p is described by the vector field $G(x,y,z) = (0, 0, -9.8)$. Find the work that this force does on p as it moves from $(0,0,0)$ to $(3, 9, -6)$.

17. Find the circumference of the ellipse $\frac{x^2}{9} + y^2 = 1$. Feel free to use mathematica.

18. Sketch the following vector fields.

(i) $F(x,y) = (x, y^2)$

(ii) $G(x,y,z) = (z, x, 0)$

(iii) $H(x,y) = (xy, x - y)$

19. Let $F(x,y) = (x, y^2)$. Find a flow line of F, namely a path $c(t)$ such that $c'(t) = F(c(t))$.

20. Let $F(x,y,z) = (y^2 + z^2, x^2 + z^2, x^2 + y^2)$ and $G(x,y,z) = (xy, x + yz^2, e^{xyz})$. Find the curl and divergence of both F and G. (Challenge: Find a vector field H such that $\nabla \times H = F$.)

21. Let R be the unit circle. Do the integral

$$\iint_R \frac{xy^2}{x^2+y^2+1}\,dA$$

without a calculator.

22. Switch the order of integration for the integral

$$\int_0^1 \int_0^{4y^2} xy\,dx\,dy.$$

23. Let $B = [-1,1] \times [2,3]$. Find the integral

$$\iint_B x^2 + y^2\,dA.$$

24. Let W be the region bounded by $z = x^2 + y^2$ and $z = 2 - 2x^2 - y^2$. Find the integral

$$\iiint_W x\,dV.$$

Try to use the symmetric properties of the region.

25. Let T be the triangular pyramid thing formed by the points (0,0,0), (3,0,0), (0,4,0), and (0,0,5). Find the volume of the shape by doing the integral in two different orders.

26. Let $f(x,y) = xy^2 + x^2y$. Find the integral of f along the path $c(t) = (3t-1, 4t+2)$ from $t = -1$ to $t = 2$. Find the vector line integral of ∇f along the same path.

27. Let $F(x,y,z) = (x\sin x, y\sin y, z\sin z)$. Let c be the line $c(t) = (t, 2t, 3t)$. Find the line integral

$$\int_c F \cdot ds$$

where c ranges from $(-\pi, -2\pi, -3\pi)$ to $(\pi, 2\pi, 3\pi)$.

28. Let $B = [0,1] \times [0,1]$ and ∂B be the perimeter and $F(x,y) = (xy^2, x^2y)$. Evaluate

$$\int_{\partial B} F ds.$$

29. Let D be the unit disc. Calculate the area

$$\int_D 1\,dA$$

both normally and by reversing Green's theorem. How do you know what direction to go around the circle in Green's theorem.